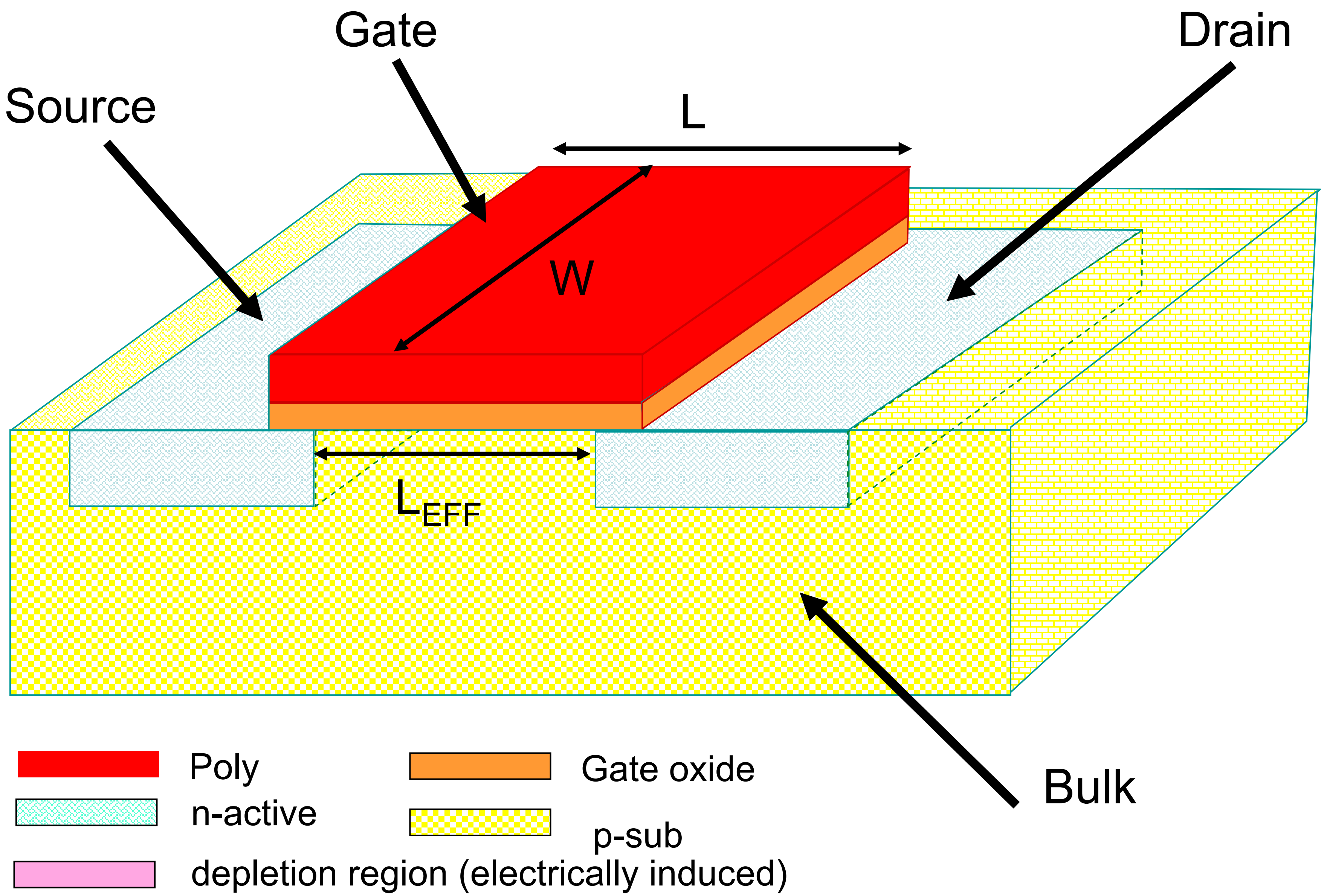
EE 230
Lecture 33

Nonlinear Circuits and Nonlinear Devices

- Diode
- BJT
- MOSFET

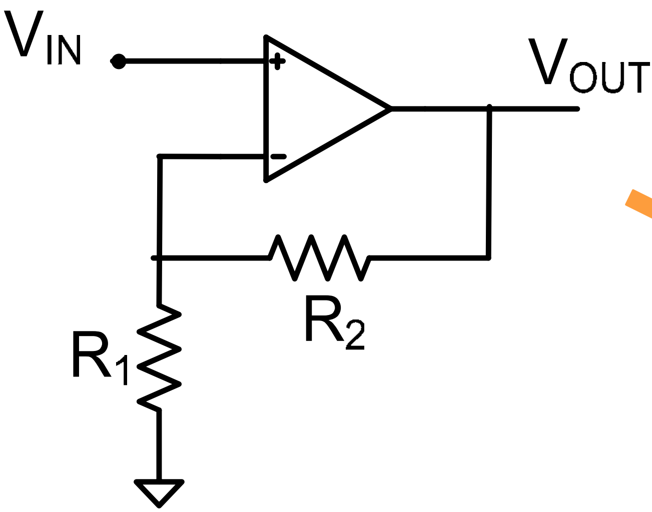
Review from Last Time:

n-Channel MOSFET

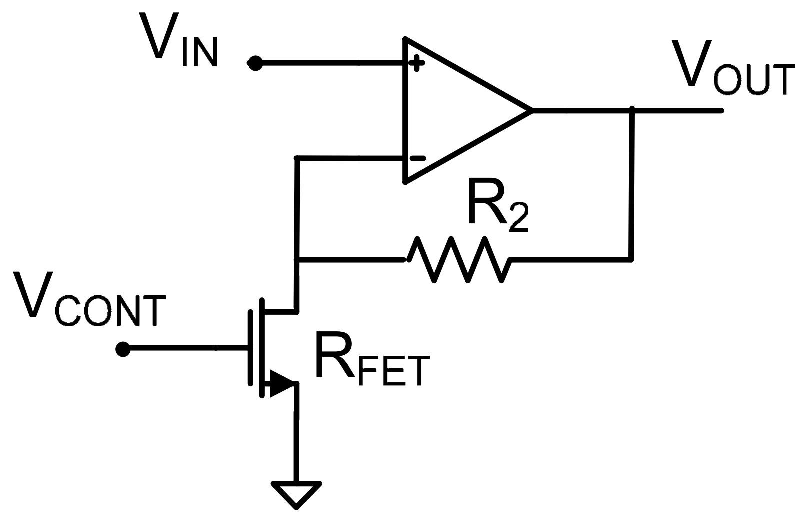


Review from Last Time:

Voltage Variable Resistor



$$A_V = 1 + \frac{R_2}{R_1}$$



$$A_V = 1 + \frac{R_2}{R_{FET}}$$

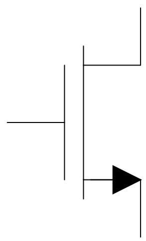
$$R_{FET} \cong \frac{1}{V_{GS} - V_T}\left(\frac{L}{\mu C_{OX} W}\right)$$

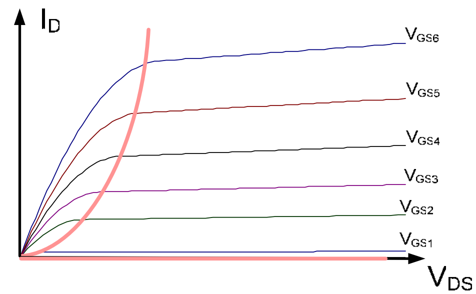
Applications include Automatic Gain Control (AGC)

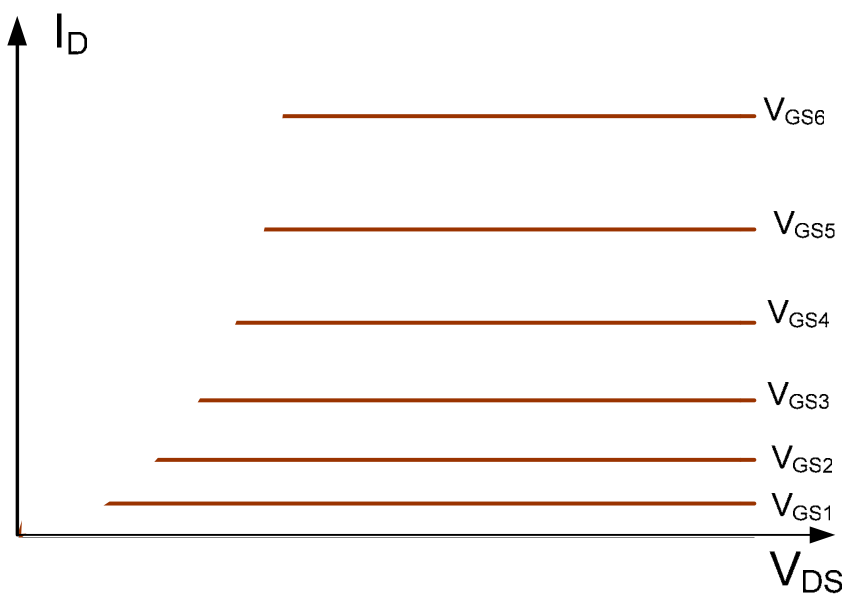
Review from Last Time:

MOS Transistor Models

simplifications







$$I_G = 0$$

$$I_D = \left(\frac{\mu C_{OX} W}{2L}\right)(V_{GS} - V_T)^2$$

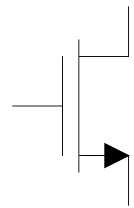
Saturation

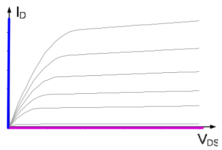
With λ=0

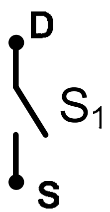
Saturation Region Model – good enough for many analog applications

Review from Last Time:

MOS Transistor Models (Summary)

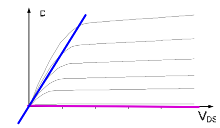


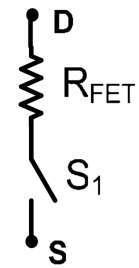




S_1 open for $V_{GS}<V_T$

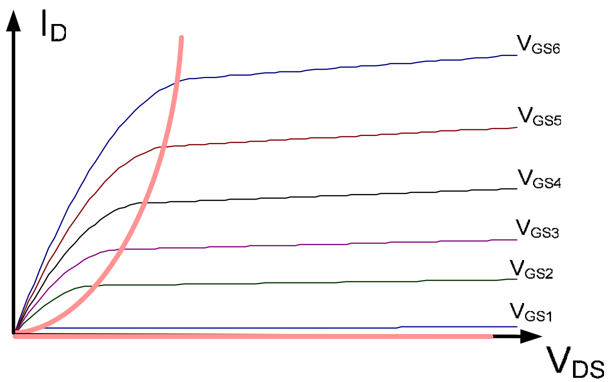
S_1 closed for $V_{GS}>V_T$

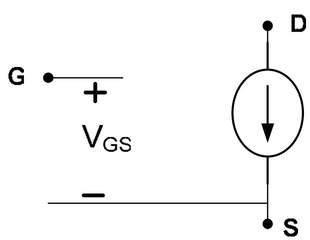




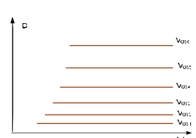
S_1 closed for $V_{GS}>V_T$

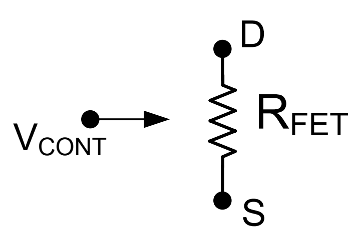
S_1 open $V_{GS}<V_T$

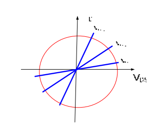




$$\left(\frac{\mu C_{ox} W}{2L}\right)(V_{GS}-V_T)^2(1+\lambda V_{DS})$$

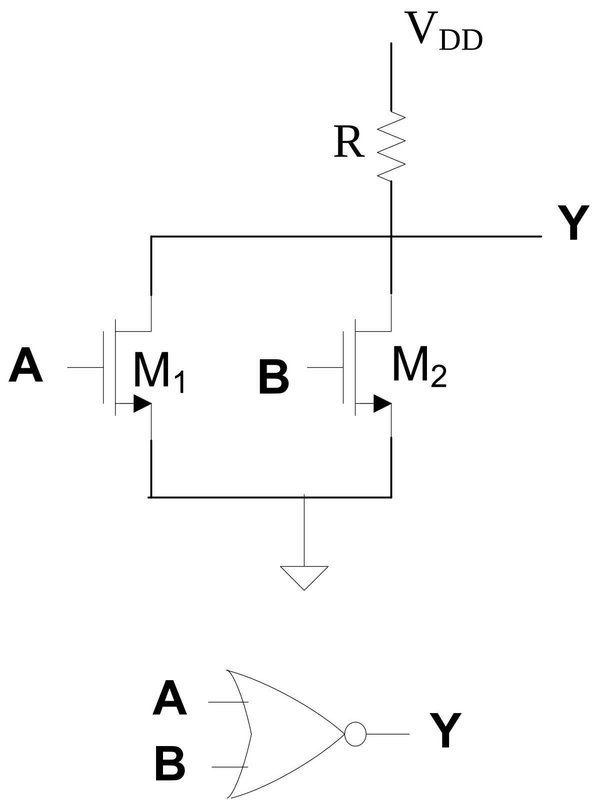


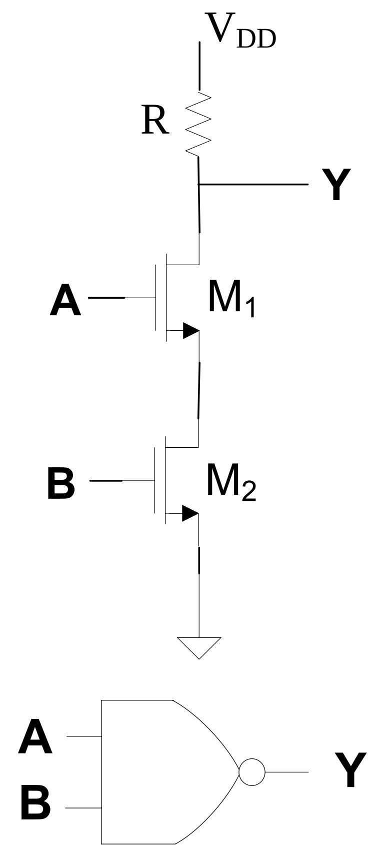




Review from Last Time:

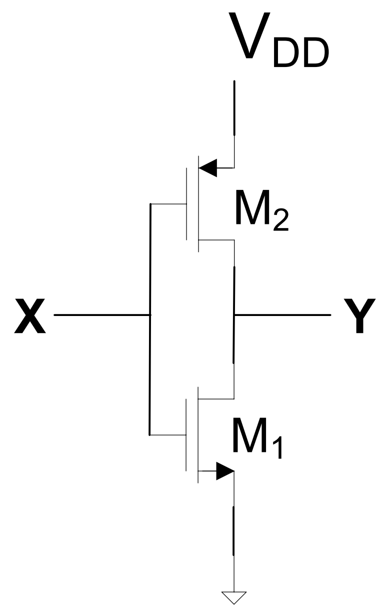
MOS Transistor Applications (Digital Circuits)





- Can be extended to arbitrary number of inputs
- But the resistor is not practically available in most processes and static power dissipation is too high

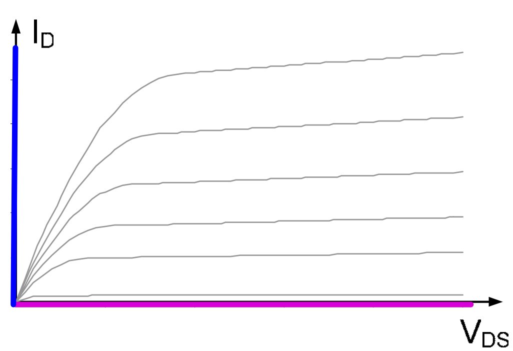
MOS Transistor Applications (Digital Circuits)

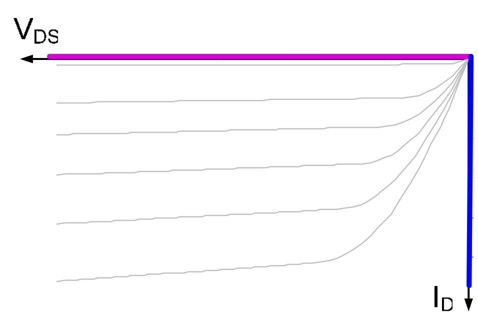


Assume "1" ~ $V_H = V_{DD}$

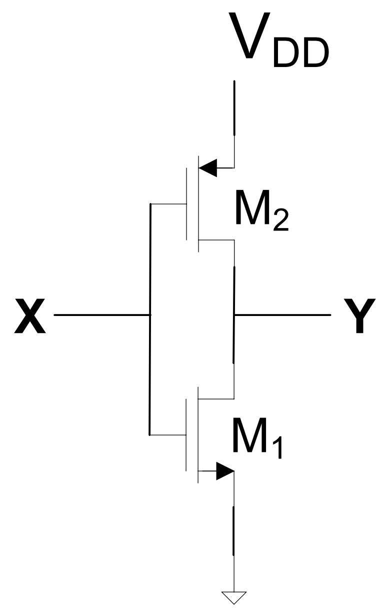
Assume "0" ~ $V_L = 0V$

MOSFET Models





MOS Transistor Applications (Digital Circuits)



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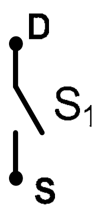
MOSFET Models

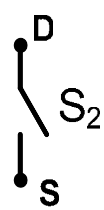


S_1 open for $V_{GS1} < V_{T1}$

S_1 closed for $V_{GS1} > V_{T1}$

Assume $V_{T1} \sim V_{DD}/5$

n-channel device



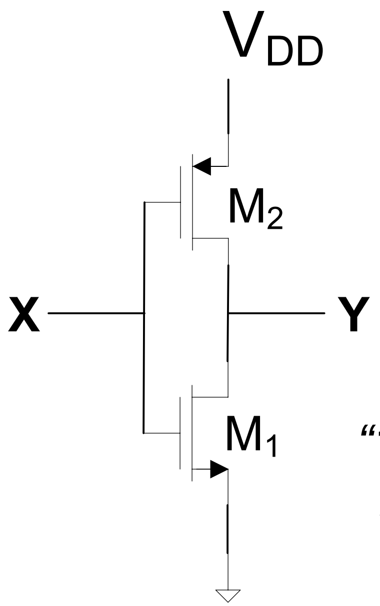
S_2 open for $V_{GS2} > V_{T2}$

S_2 closed for $V_{GS2} < V_{T2}$

Assume $V_{T2} \sim - V_{DD}/5$

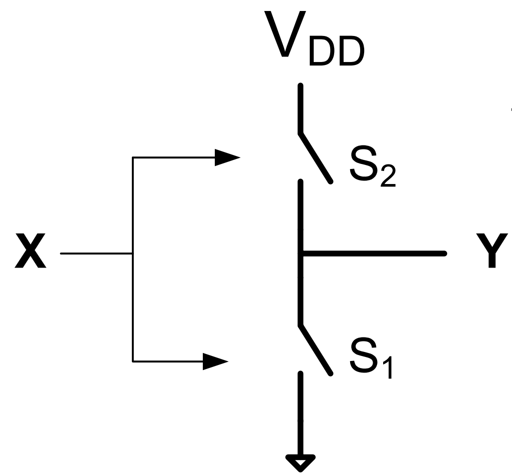
p-channel device

MOS Transistor Applications
(Digital Circuits)



"1" ~ $V_H = V_{DD}$

"0" ~ $V_L = 0V$



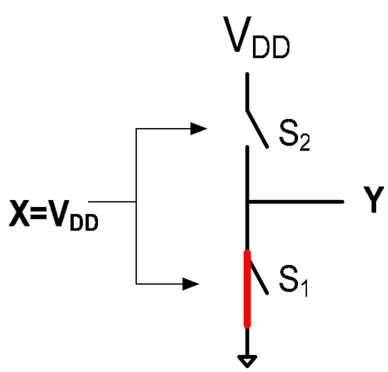
S_2 open for $V_{GS2}>V_{T2}$

S_2 closed for $V_{GS2}<V_{T2}$

S_1 open for $V_{GS1}<V_{T1}$

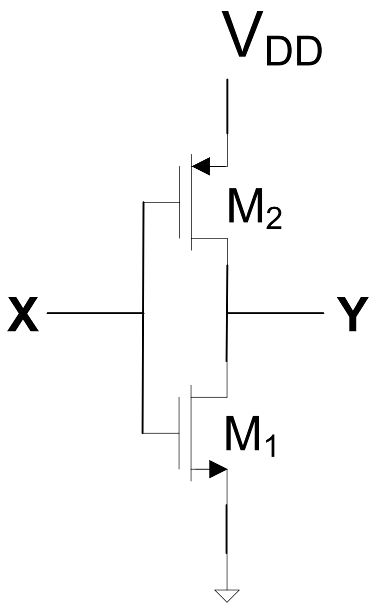
S_1 closed for $V_{GS1}>V_{T1}$

If $X=V_{DD}$*, then* $V_{GS1}=V_{DD}>V_{T1}$, $V_{GS2}=0 > V_{T2}$ $\Rightarrow$ S_1 *closed,* S_2 *open*



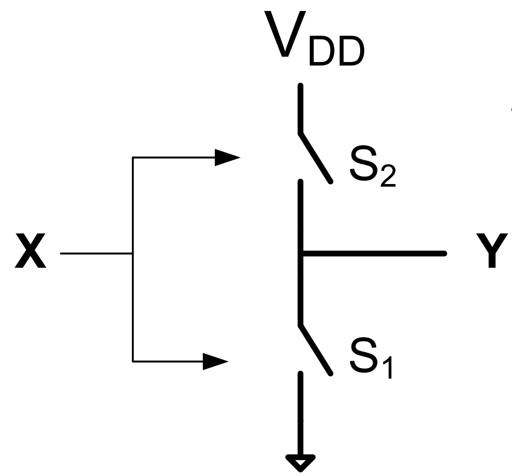
$Y = 0V$~"0"

MOS Transistor Applications (Digital Circuits)



"1" ~ $V_H = V_{DD}$

"0" ~ $V_L = 0V$



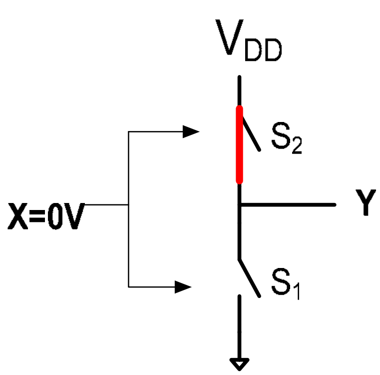
S_2 open for $V_{GS2}>V_{T2}$

S_2 closed for $V_{GS2}<V_{T2}$

S_1 open for $V_{GS1}<V_{T1}$

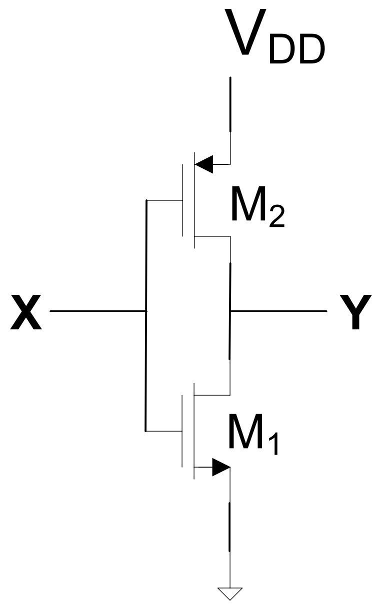
S_1 closed for $V_{GS1}>V_{T1}$

If X=0V , then $V_{GS1}=0V<V_{T1}$, $V_{GS2}=-V_{DD} < V_{T2}$ → *S_2 closed, S_1 open*



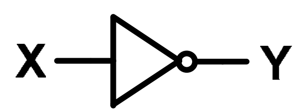
$Y = V_{DD}$~"1"

MOS Transistor Applications (Digital Circuits)



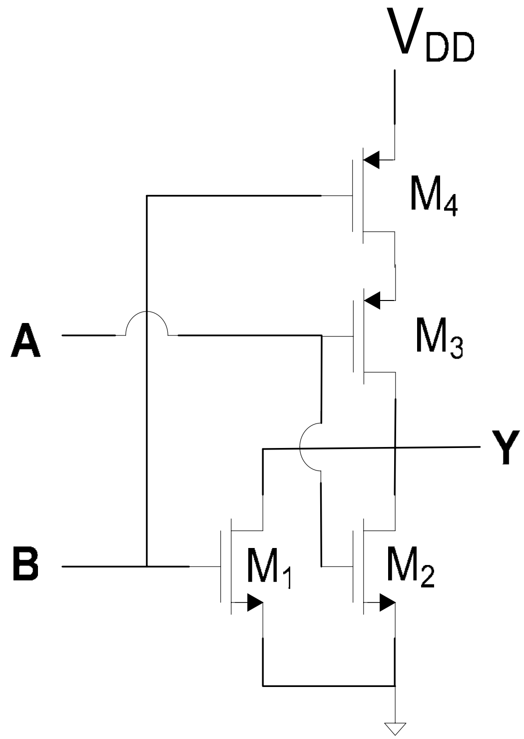
X	Y
0	1
1	0

Truth Table



Performs as a digital inverter

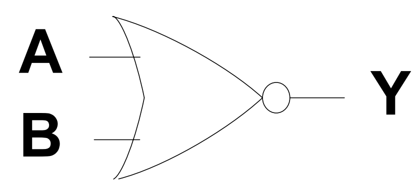
MOS Transistor Applications (Digital Circuits)



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

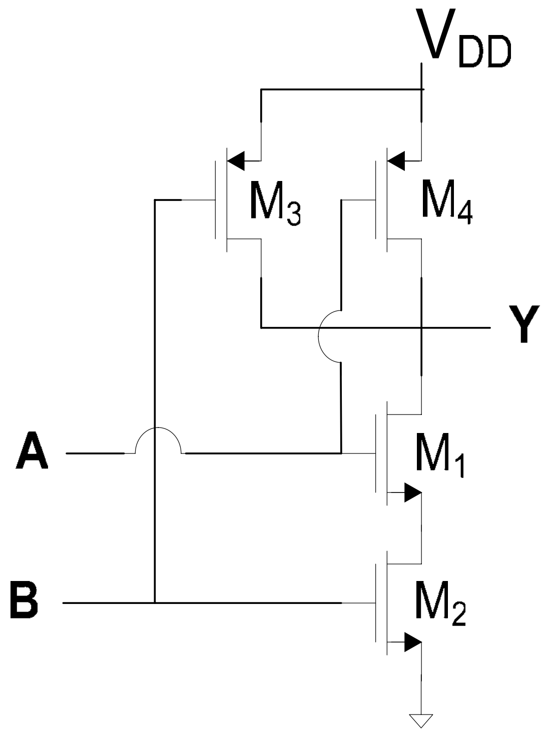
Truth Table

Performs as a 2-input NOR Gate



Can be easily extended to an n-input NOR Gate

MOS Transistor Applications (Digital Circuits)

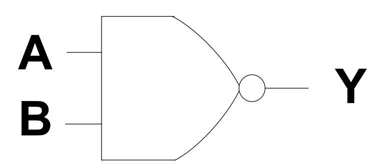


A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

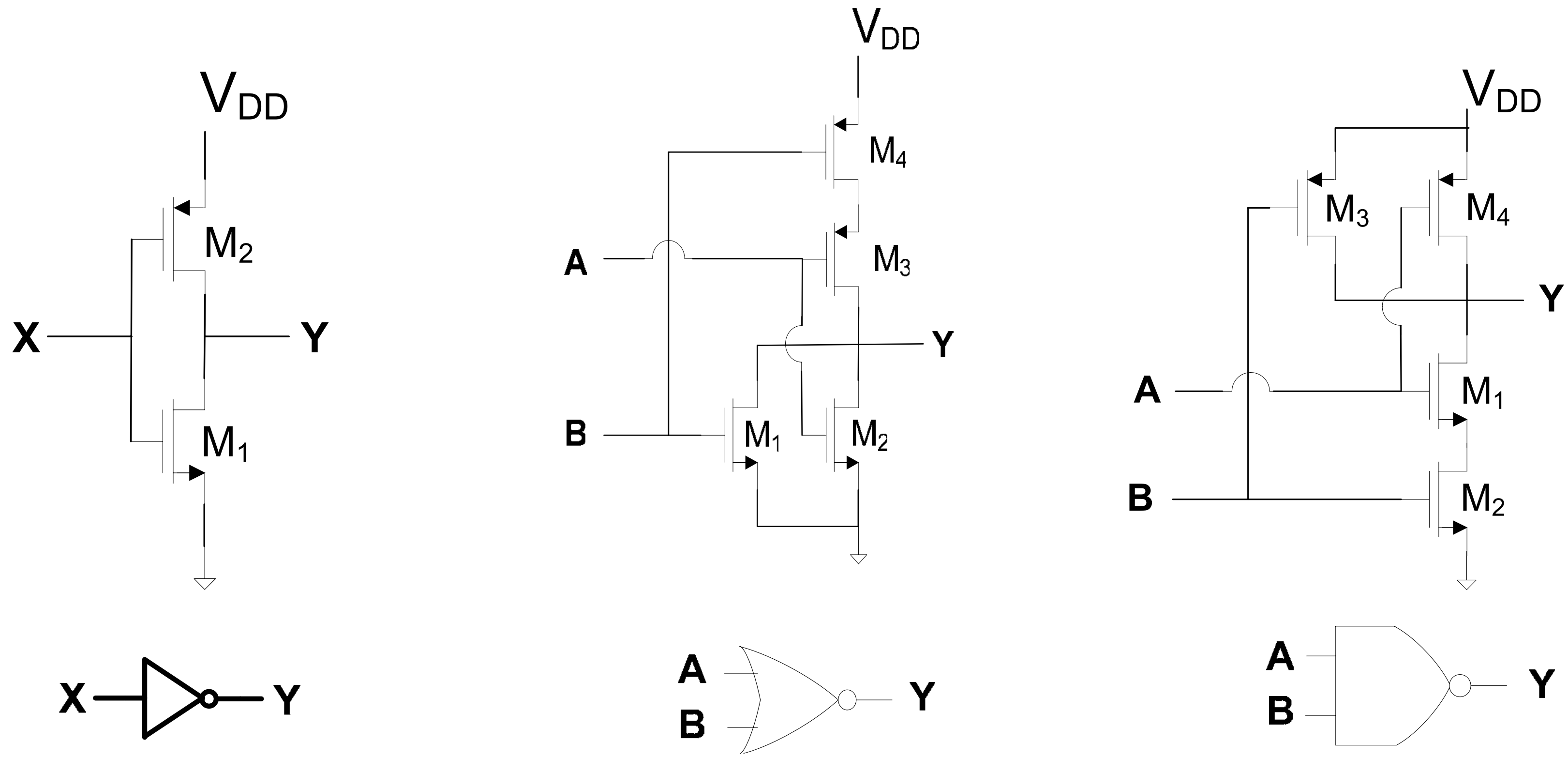
Truth Table

Performs as a 2-input NAND Gate

Can be easily extended to an n-input NAND Gate

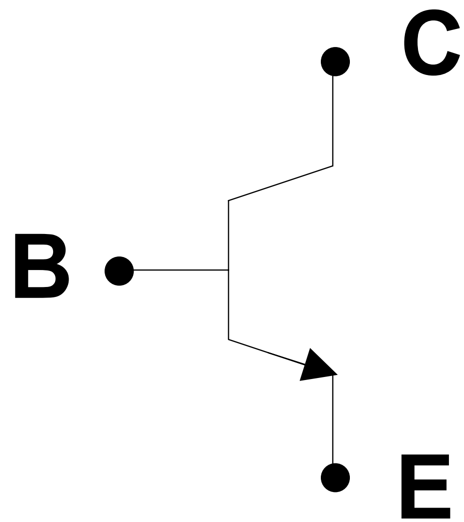


MOS Transistor Applications (Digital Circuits)



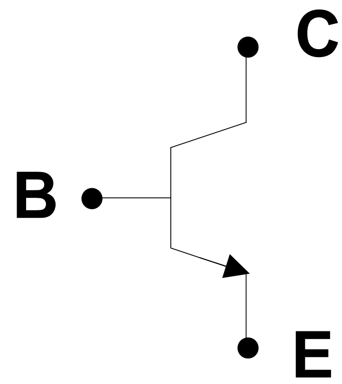
- Termed CMOS Logic
- Widely used in industry today (millions of transistors in many ICs using this logic
- Almost never used as discrete devices

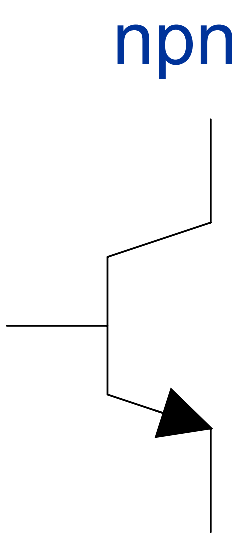
Bipolar Transistor

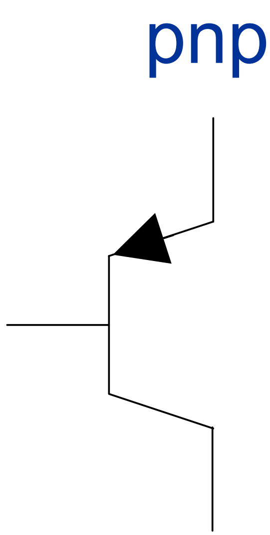


B: Base
C: Collector
E: Emitter

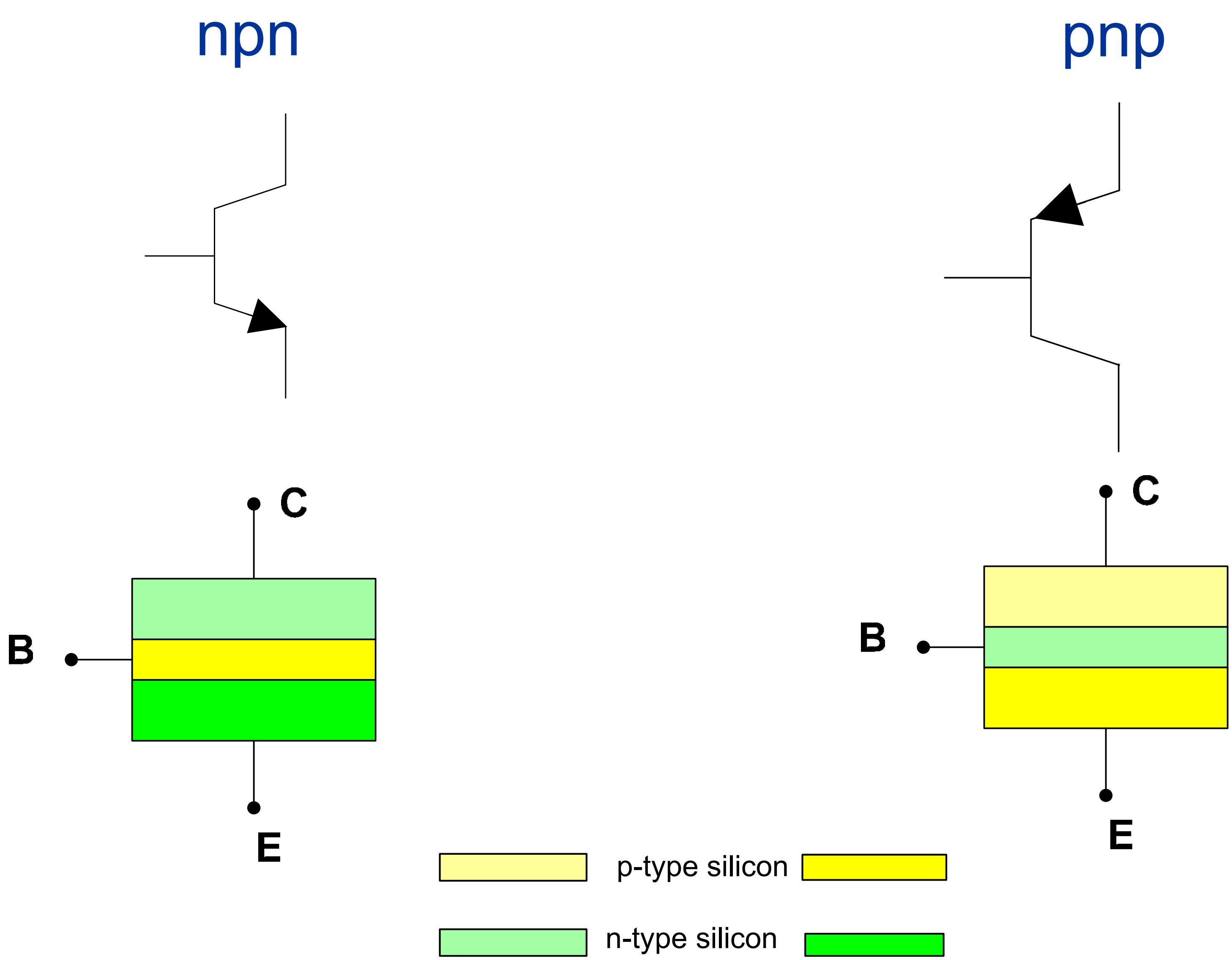
Bipolar Transistor



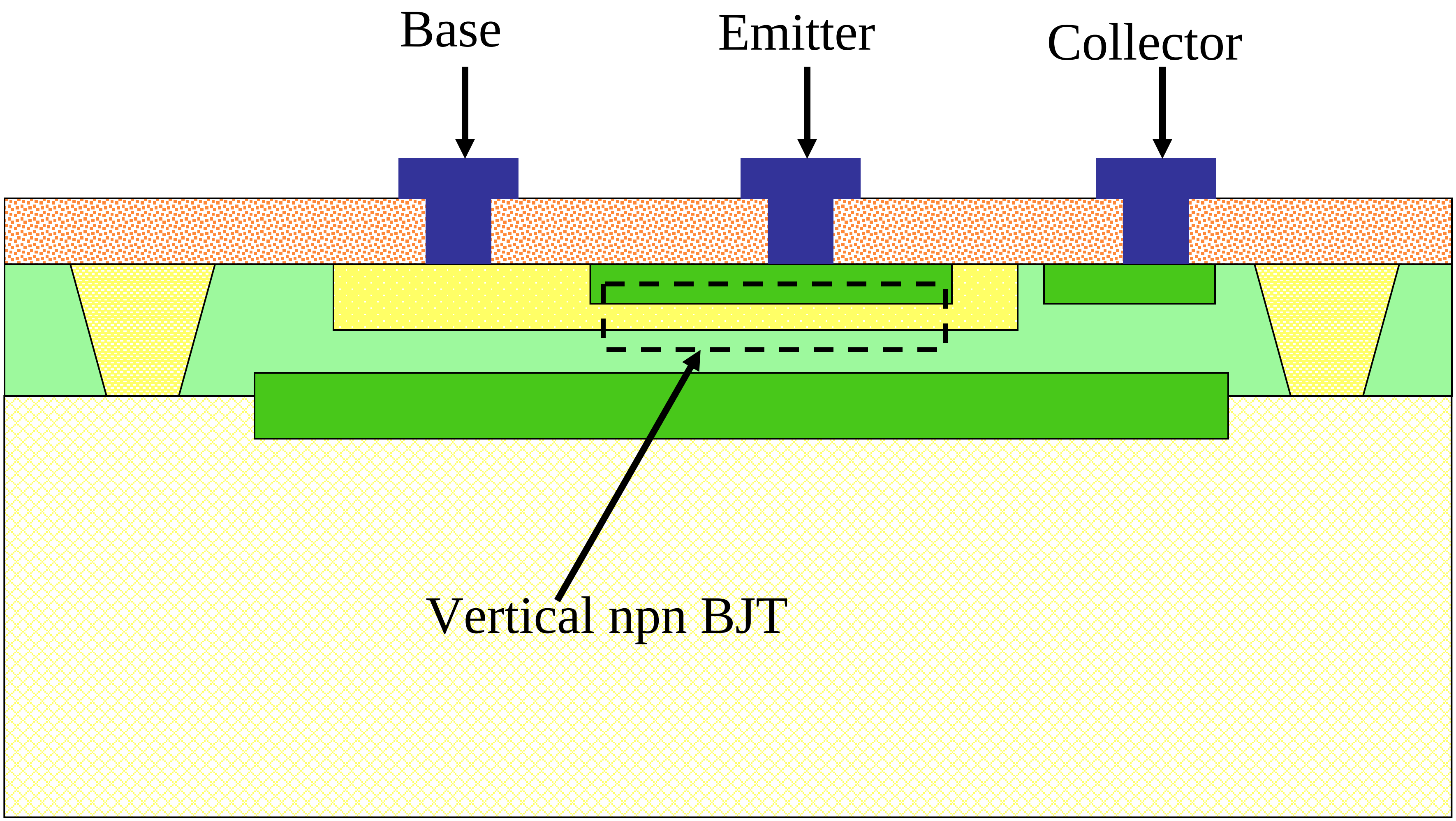




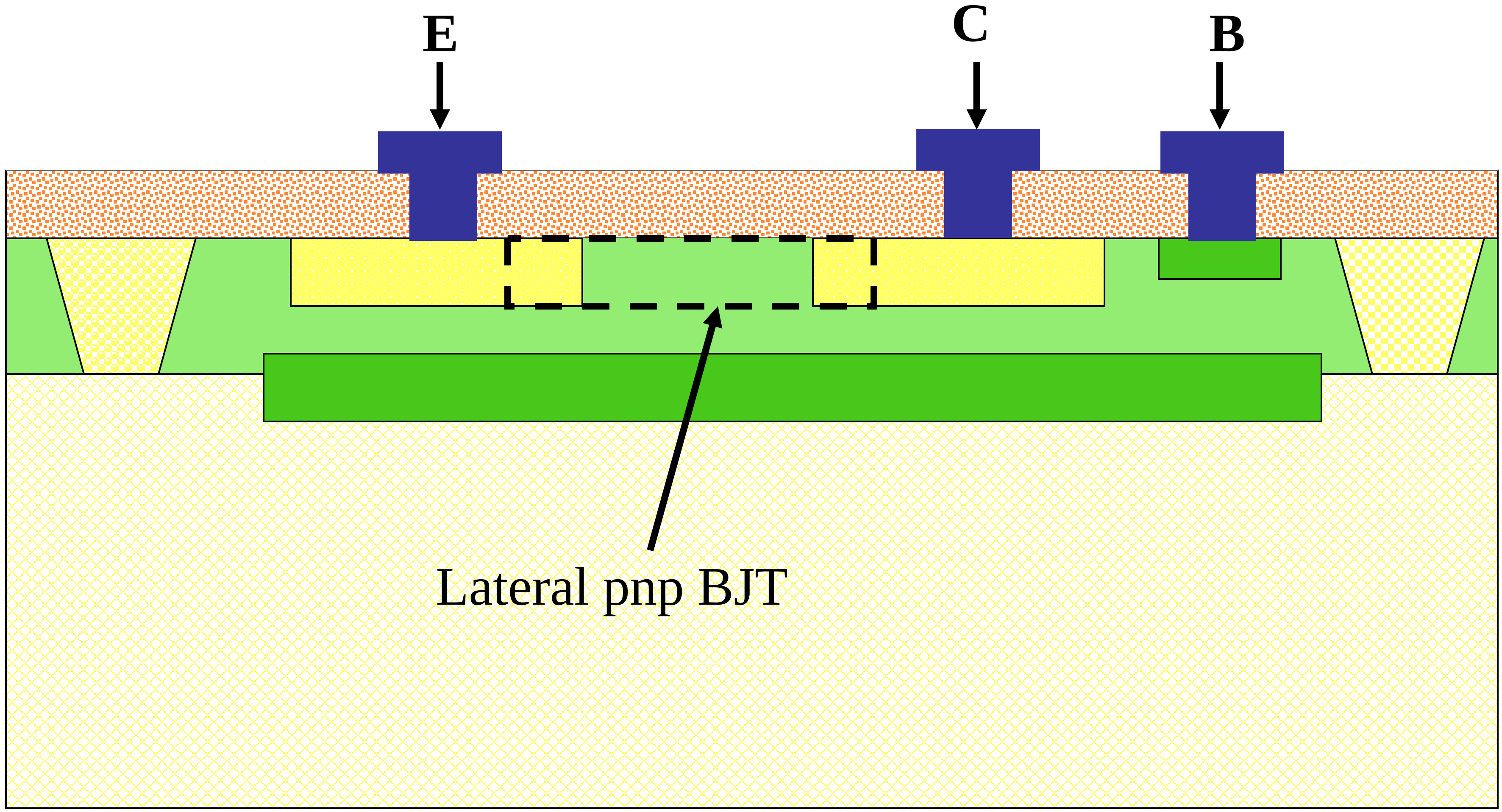
Bipolar Transistor



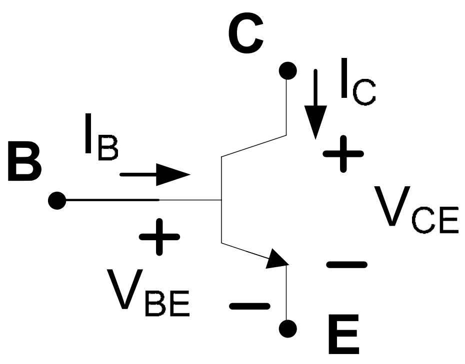
Vertical npn BJT



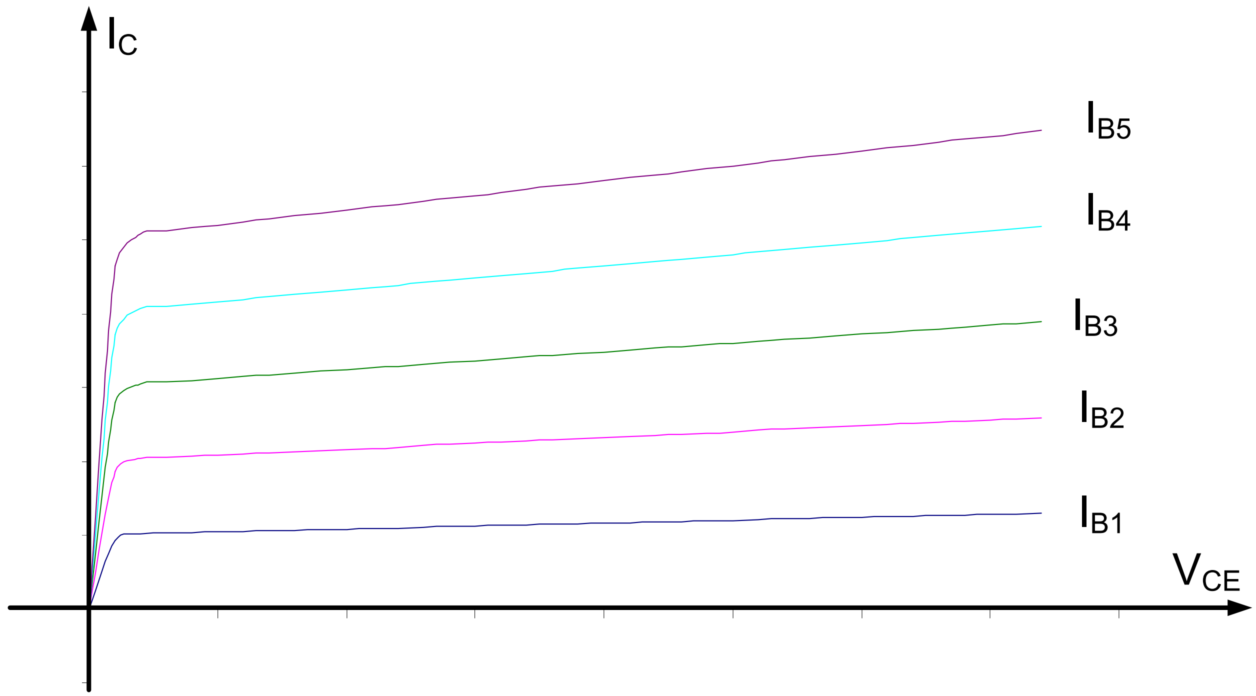
Lateral pnp BJT



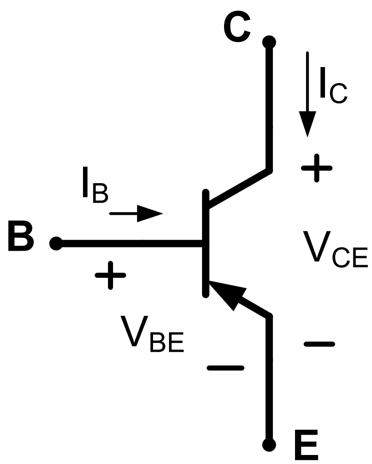
Bipolar Transistor



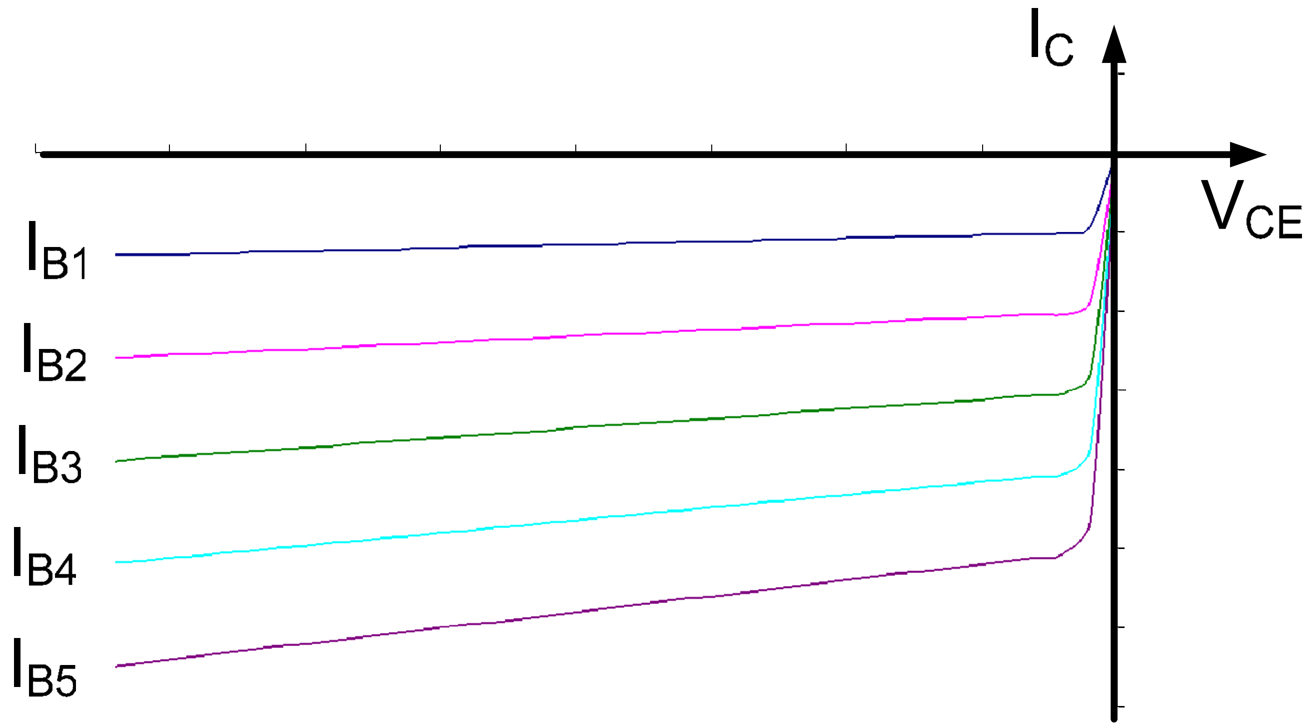
npn



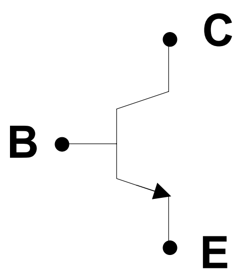
Bipolar Transistor

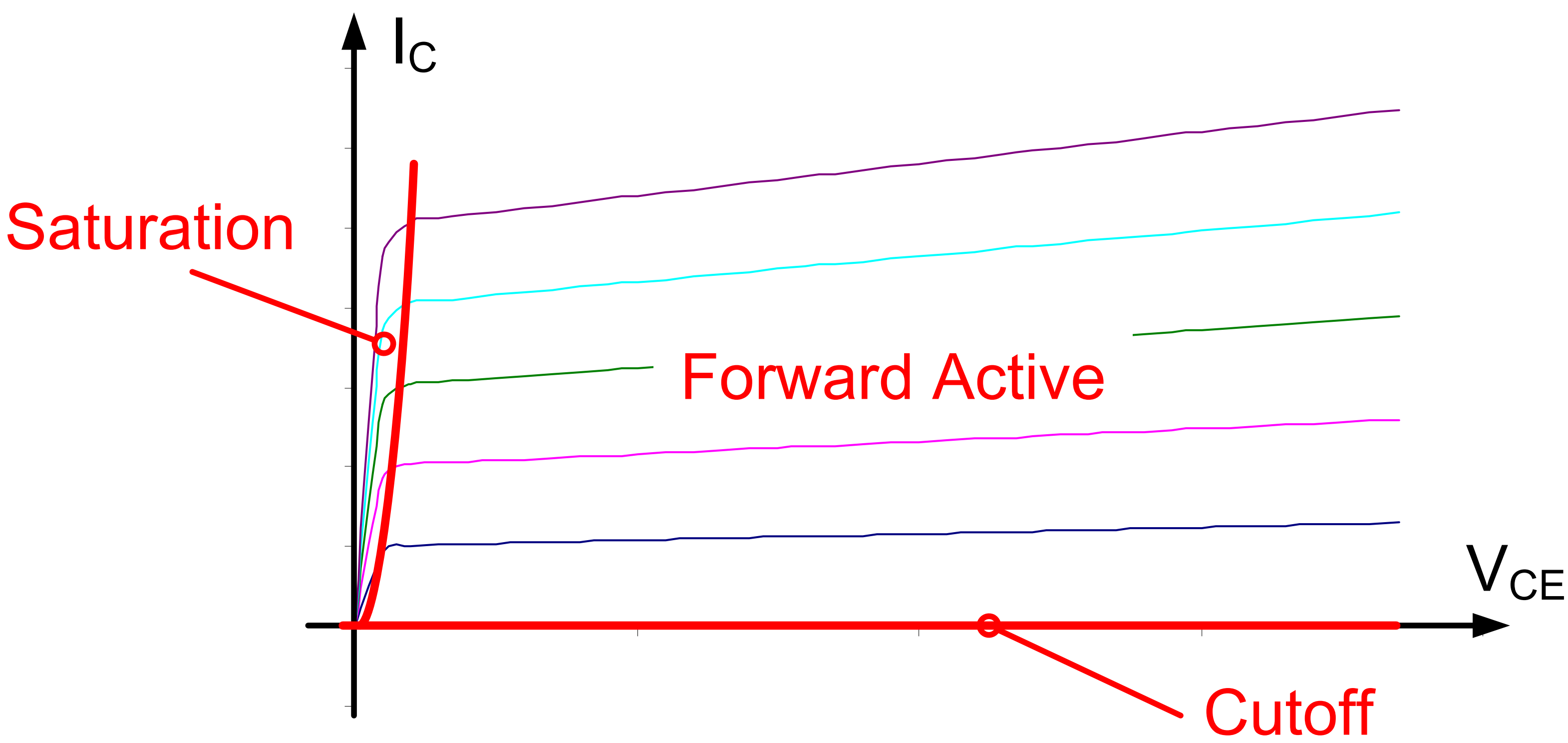


pnp

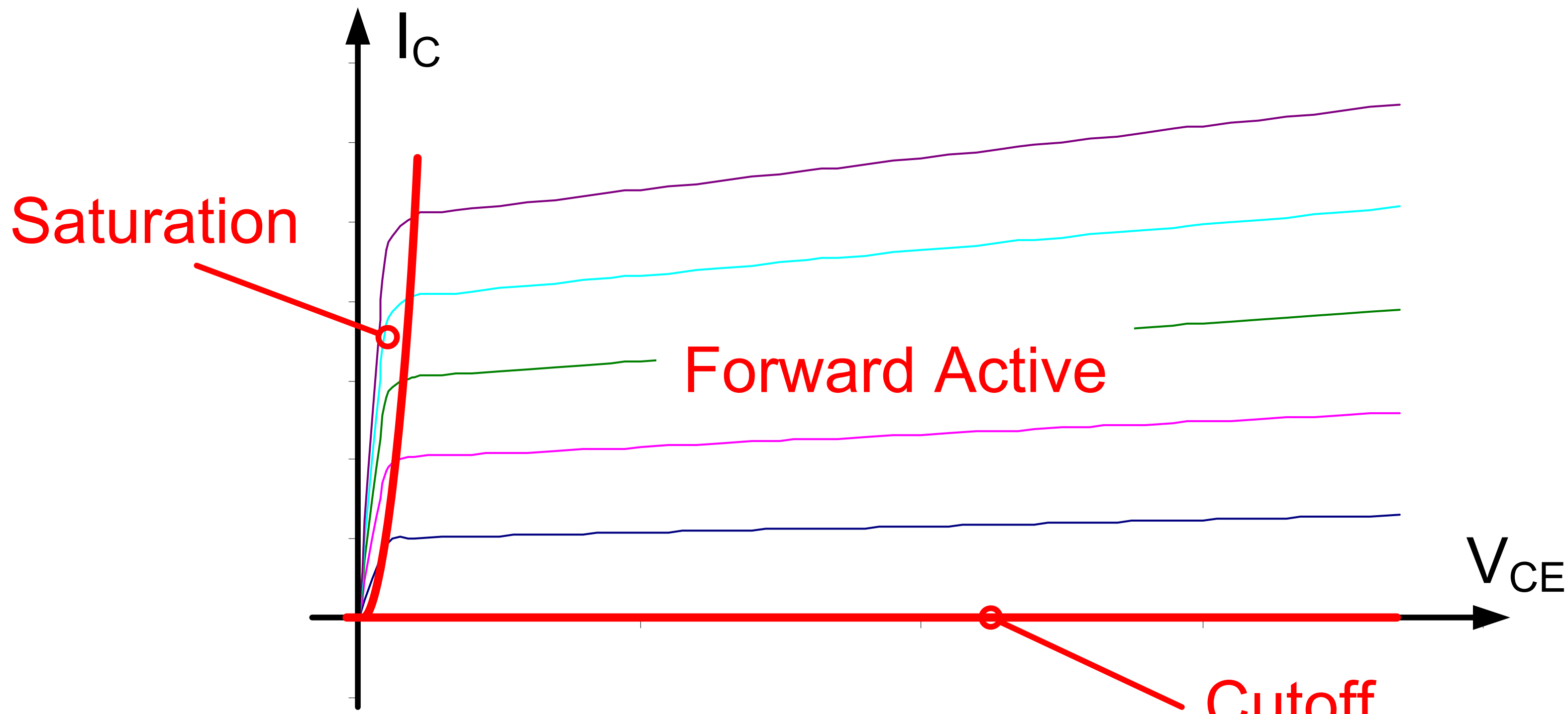


Bipolar Transistor





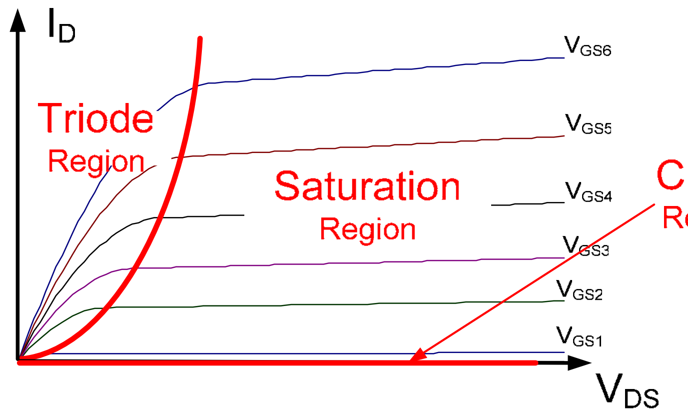
Bipolar Transistor



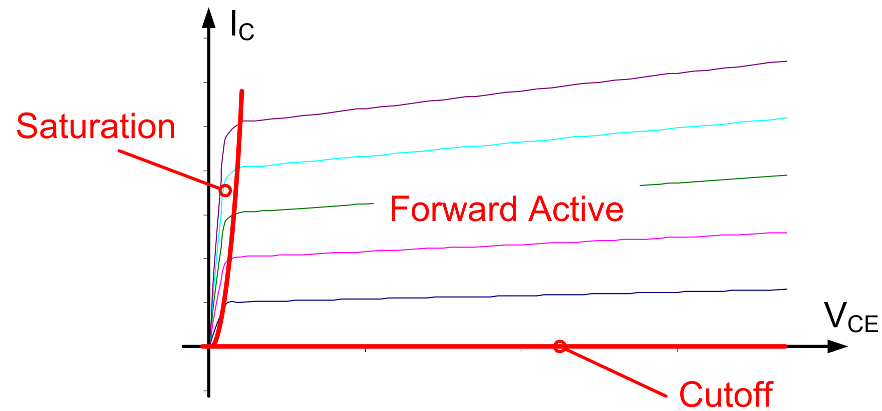
Most analog or linear applications based upon Forward Active region

Most digital applications involve Saturation and Cutoff regions and switching between these regions as the Boolean value changes states

Bipolar and MOS Region Comparisons



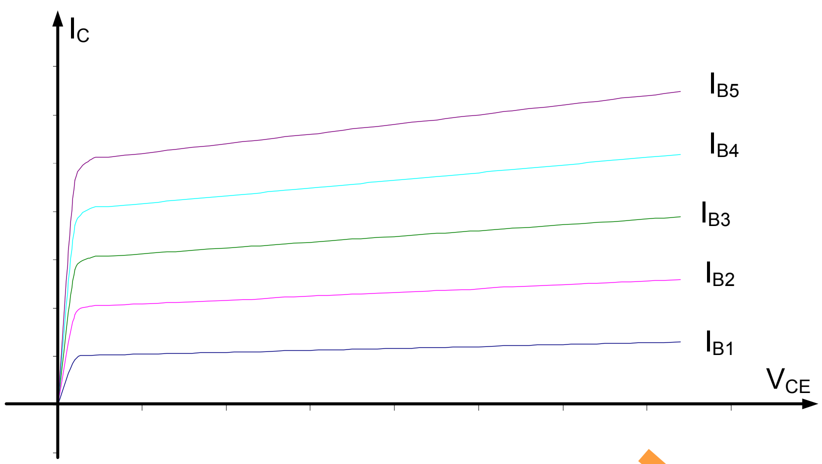
MOSFET

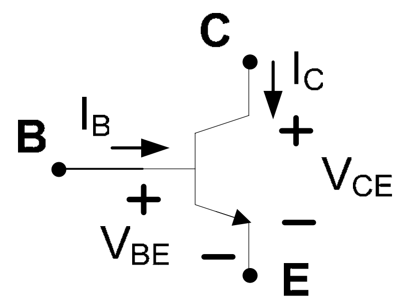


BJT

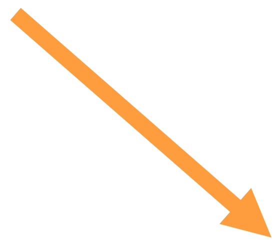
Cutoff	⟷	Cutoff
Saturation	⟷	Forward Active
Triode	⟷	Saturation

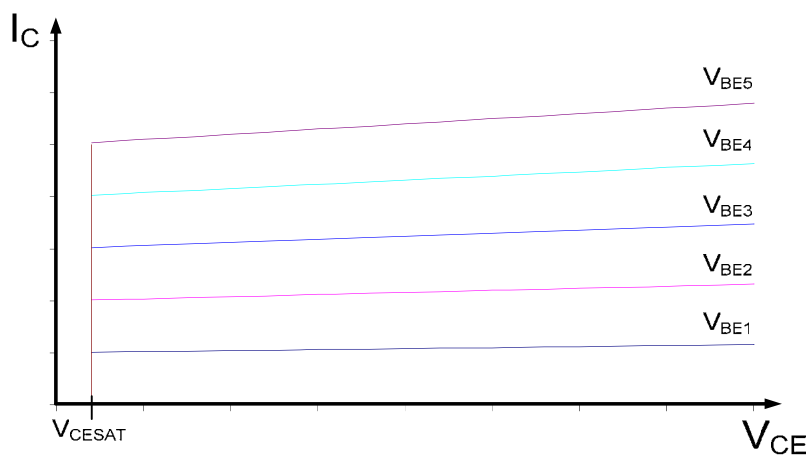
Bipolar Transistor





npn





Bipolar Transistor

Multi-Region Model

$$\mathbf{I_C = J_S A_E e^{\frac{V_{BE}}{V_t}}\left(1+\frac{V_{CE}}{V_{AF}}\right) = \beta I_B\left(1+\frac{V_{CE}}{V_{AF}}\right)}$$

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$V_t = \frac{kT}{q}$$

$V_{BE}>0.4V$

$V_{BC}<0$

Forward Active

$V_{BE}=0.7V$
$V_{CE}=0.2V$

$I_C<\beta I_B$

Saturation

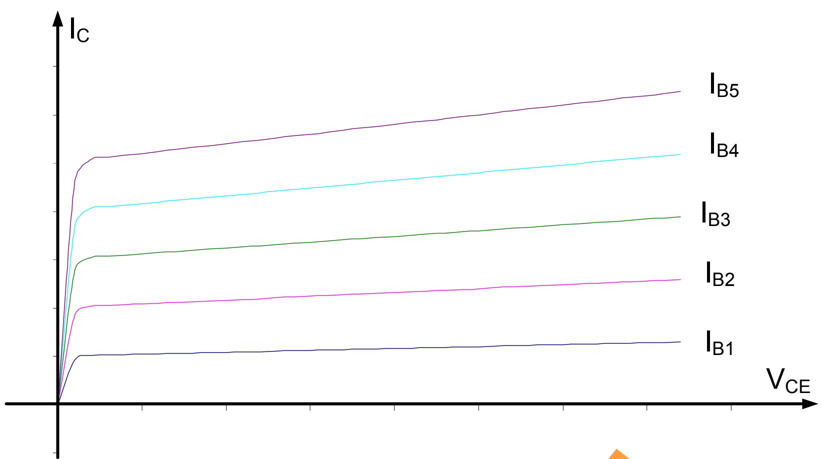
$I_C=I_B=0$

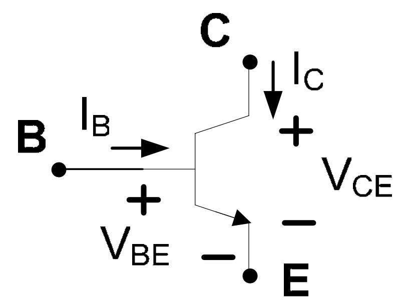
$V_{BE}<0$

$V_{BC}<0$

Cutoff

Bipolar Transistor





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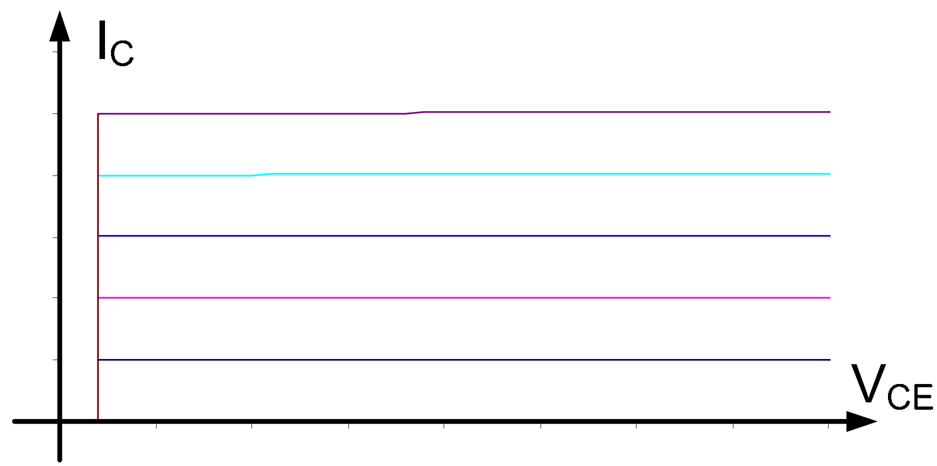

Bipolar Transistor

Simplifier Basic Multi-Region Model

$$\mathbf{I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} = \beta I_B}$$

$$\mathbf{I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}}$$

$$V_t = \frac{kT}{q}$$

$V_{BE}>0.4V$

$V_{BC}<0$

Forward Active

$V_{BE}=0.7V$
$V_{CE}=0.2V$

$I_C<\beta I_B$

Saturation

$I_C=I_B=0$

$V_{BE}<0$

$V_{BC}<0$

Cutoff

Small-signal Operation of Nonlinear Circuits

- Small-signal principles
- Example Circuit
- Small-Signal Models
- Small-Signal Analysis of Nonlinear Circuits

Small-signal Operation of Nonlinear Circuits

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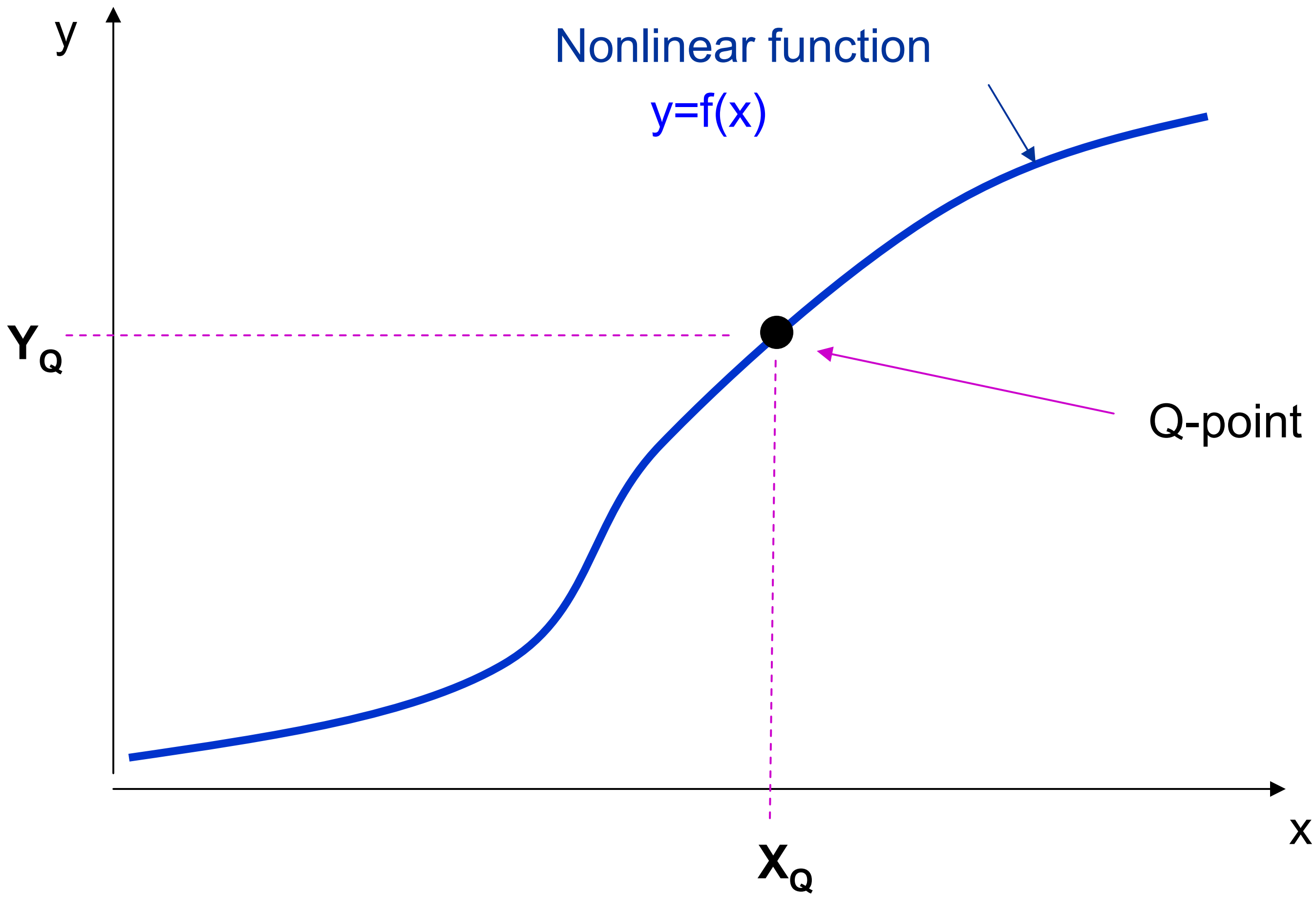
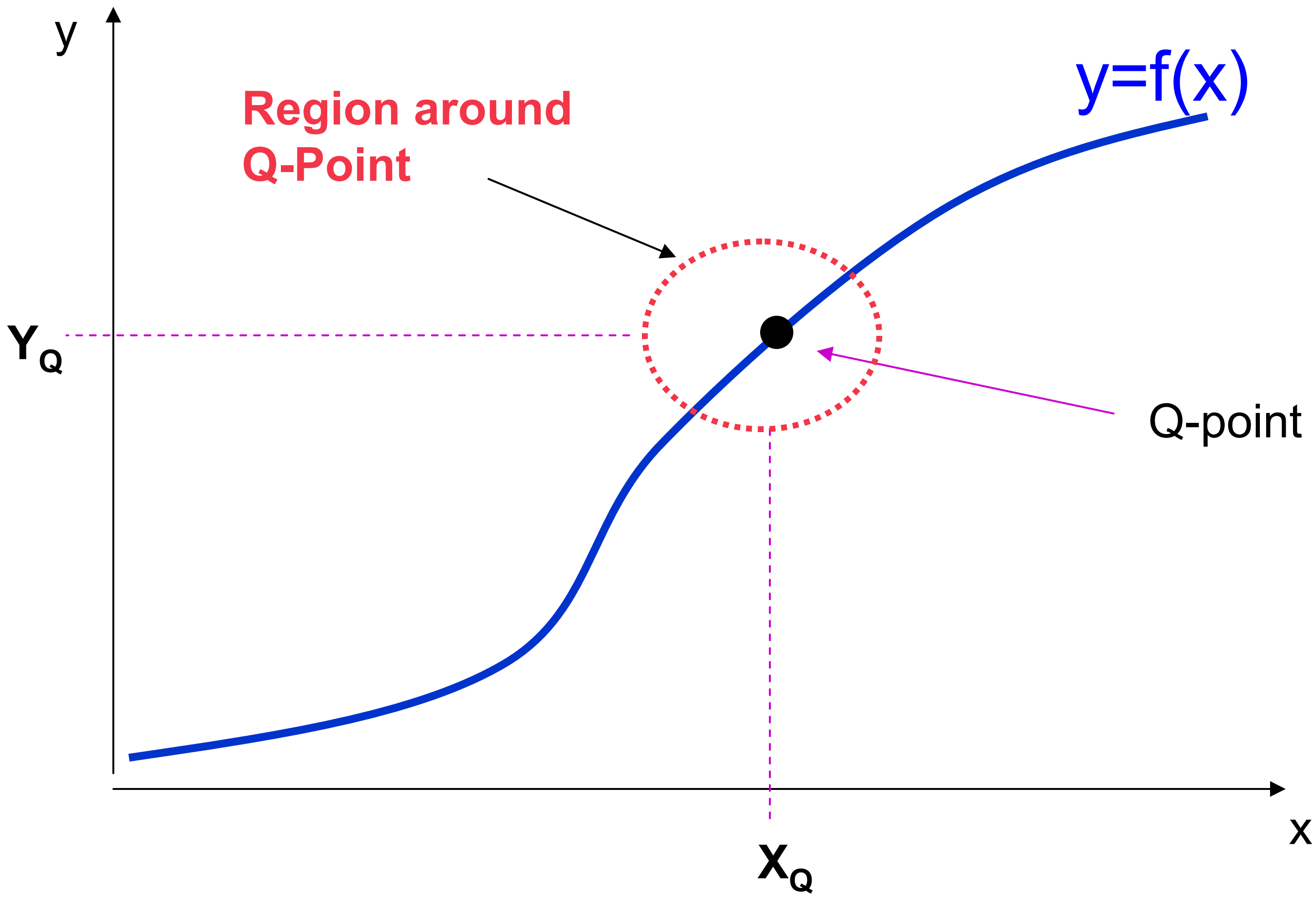
- Small-Signal Analysis of Nonlinear Circuits

Small-Signal Principle



Small-Signal Principle



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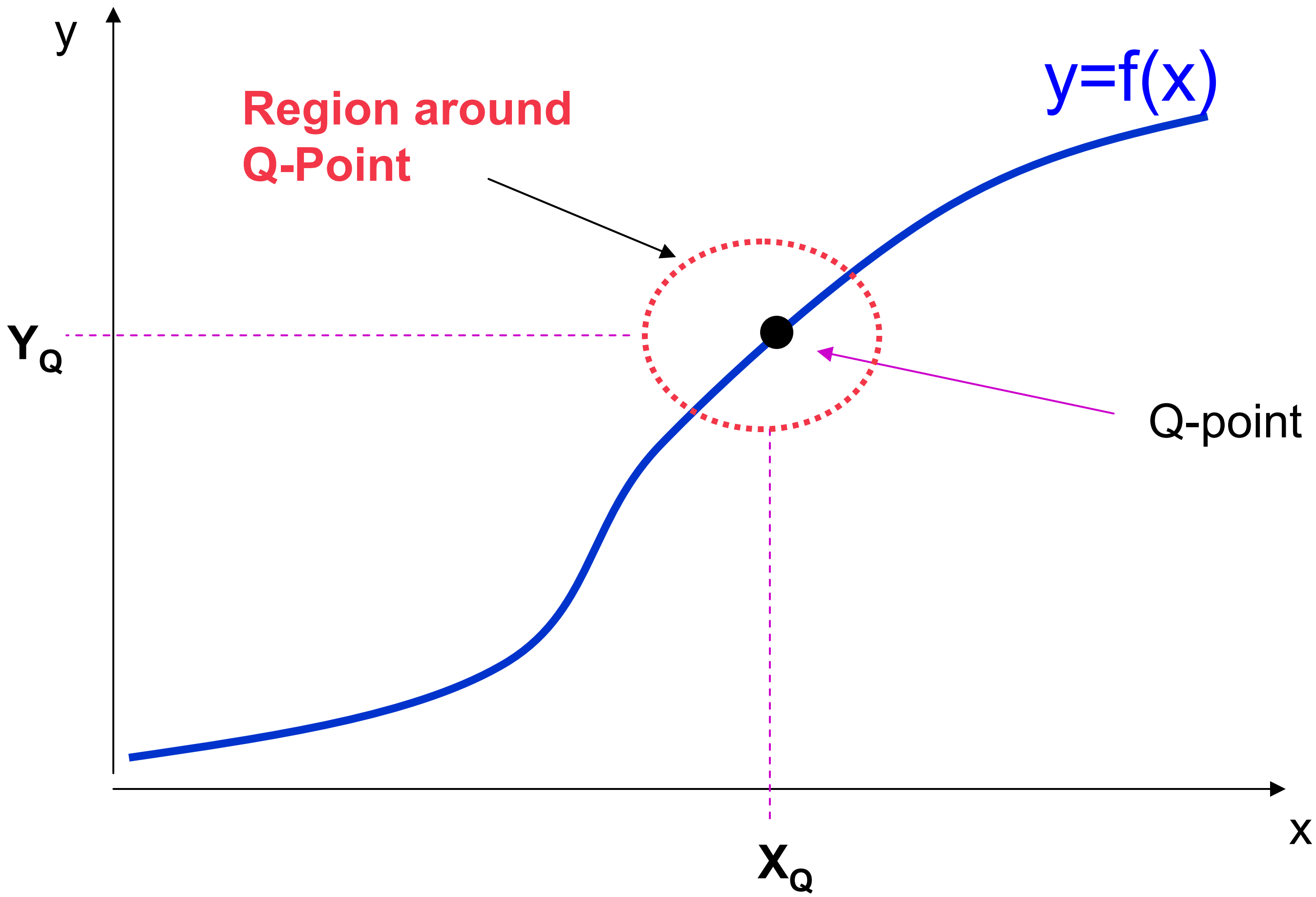
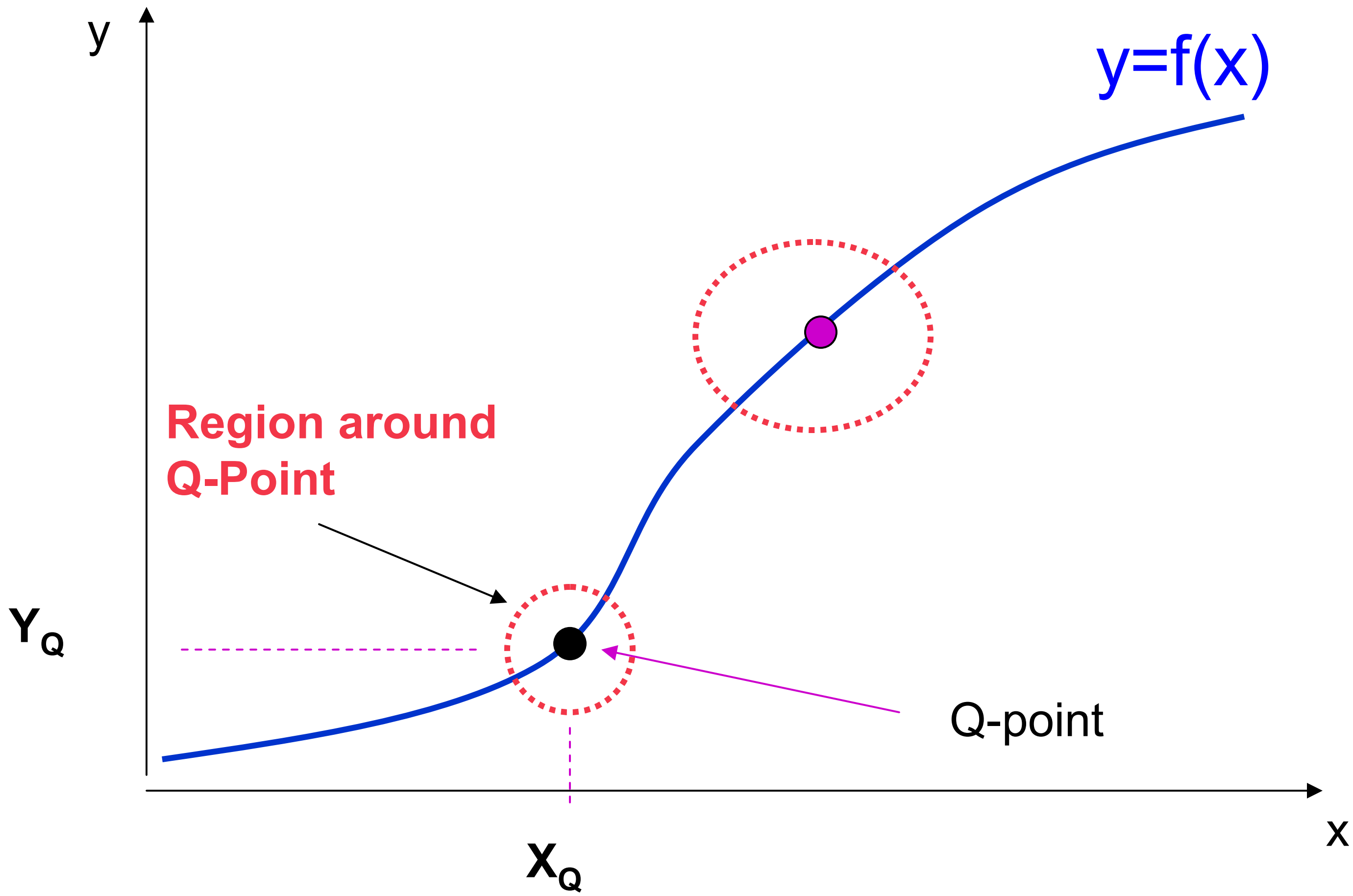


Relationship is nearly linear in a small enough region around Q-point
Region of linearity is often quite large
Linear relationship may be different for different Q-points

Small-Signal Principle



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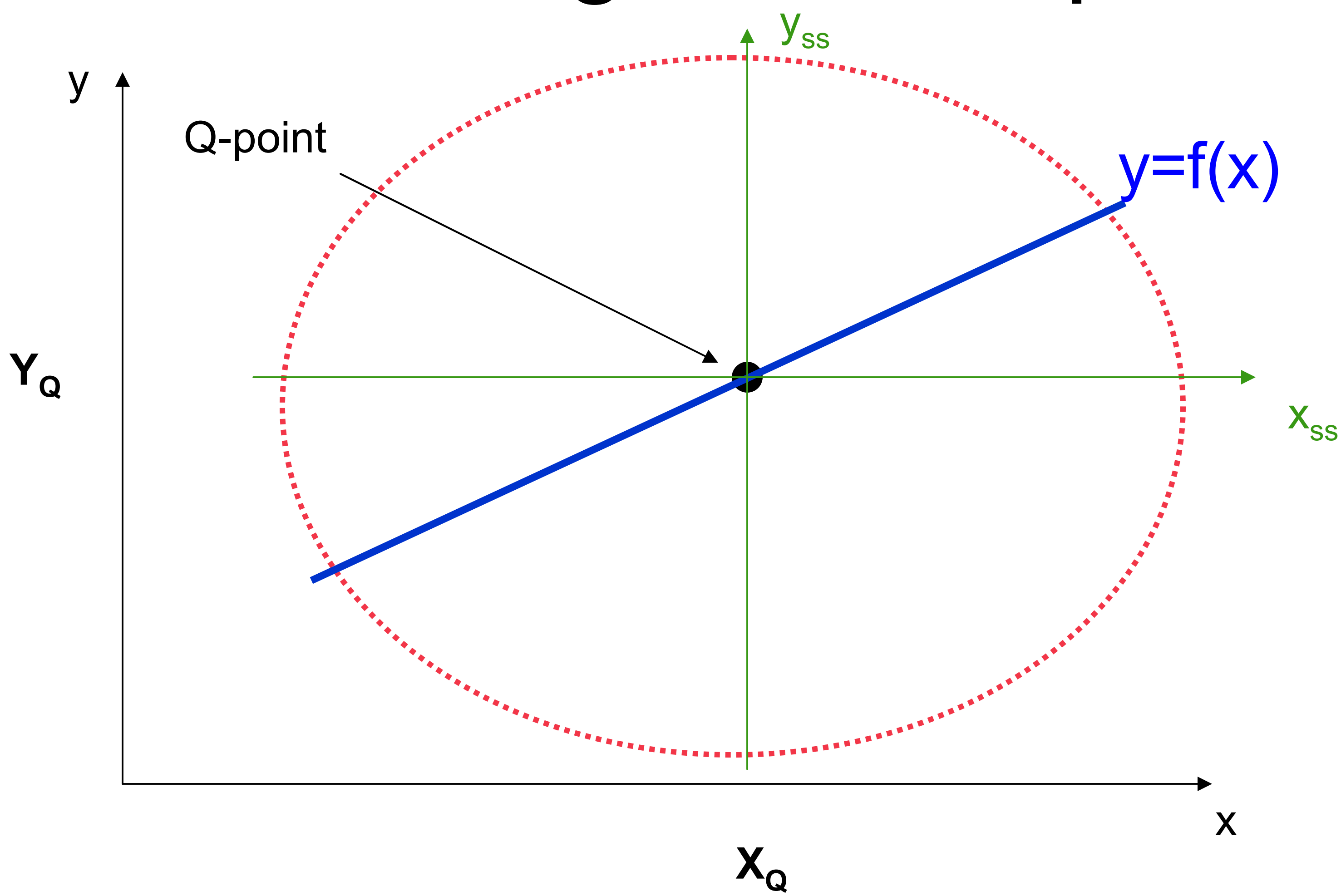
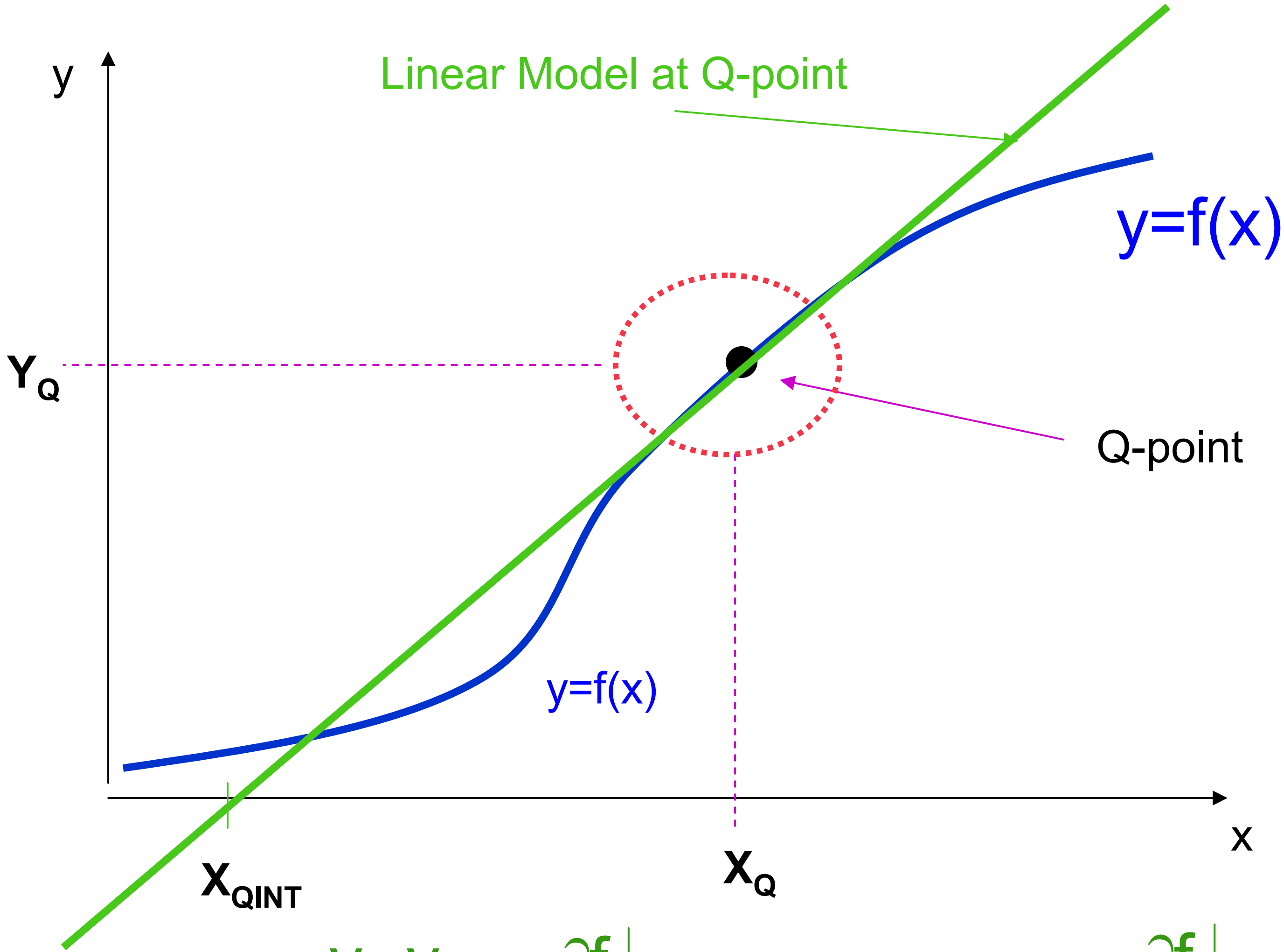


- Device behaves linearly in neighborhood of Q-point
- Can be characterized in terms of a small-signal coordinate system

Small-Signal Principle



$$\frac{y-y_Q}{x-x_Q}=\left.\frac{\partial f}{\partial x}\right|_{x=x_Q} \rightarrow y-y_Q=\left.\frac{\partial f}{\partial x}\right|_{x=x_Q}(x-x_Q)$$

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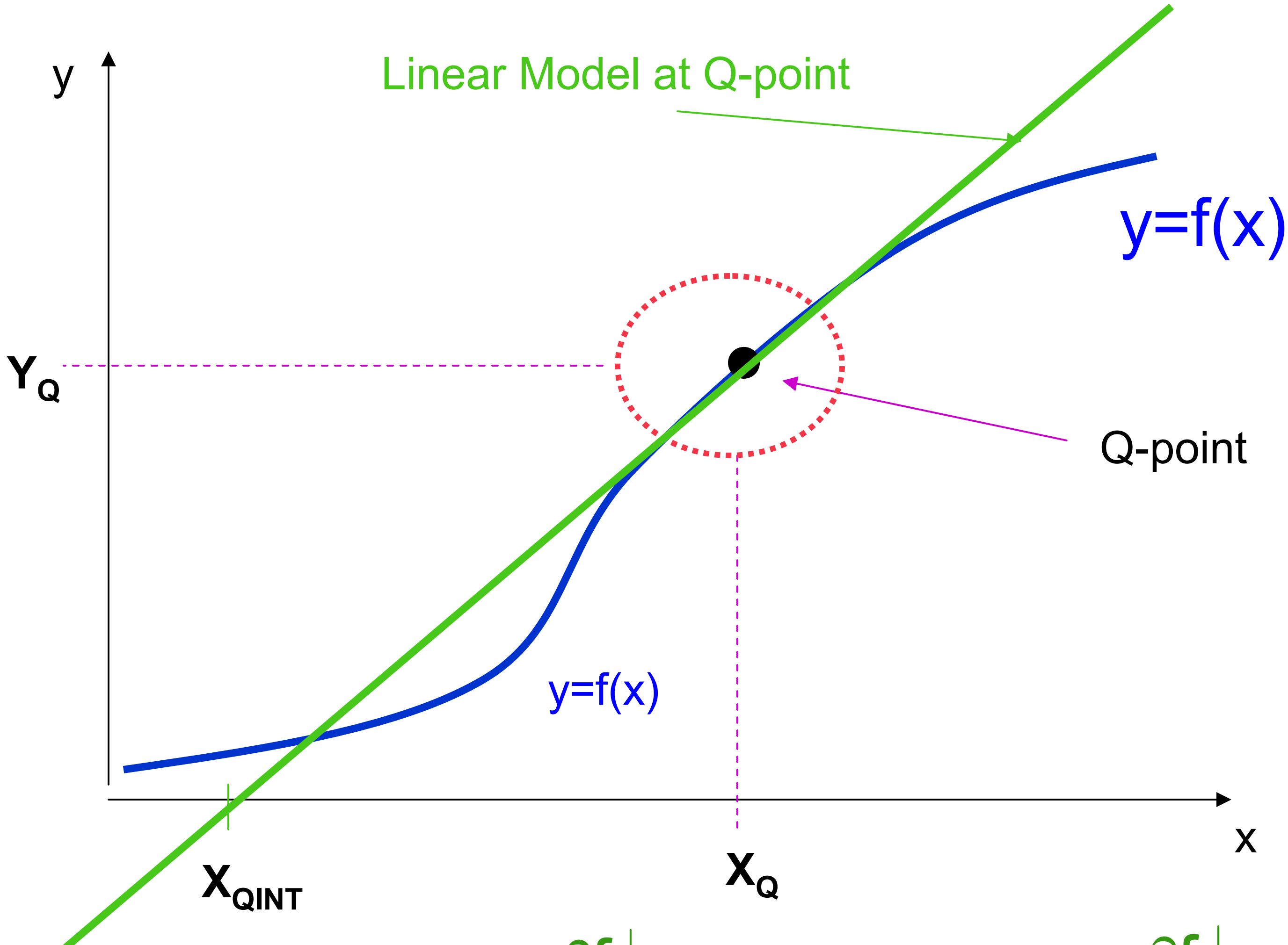

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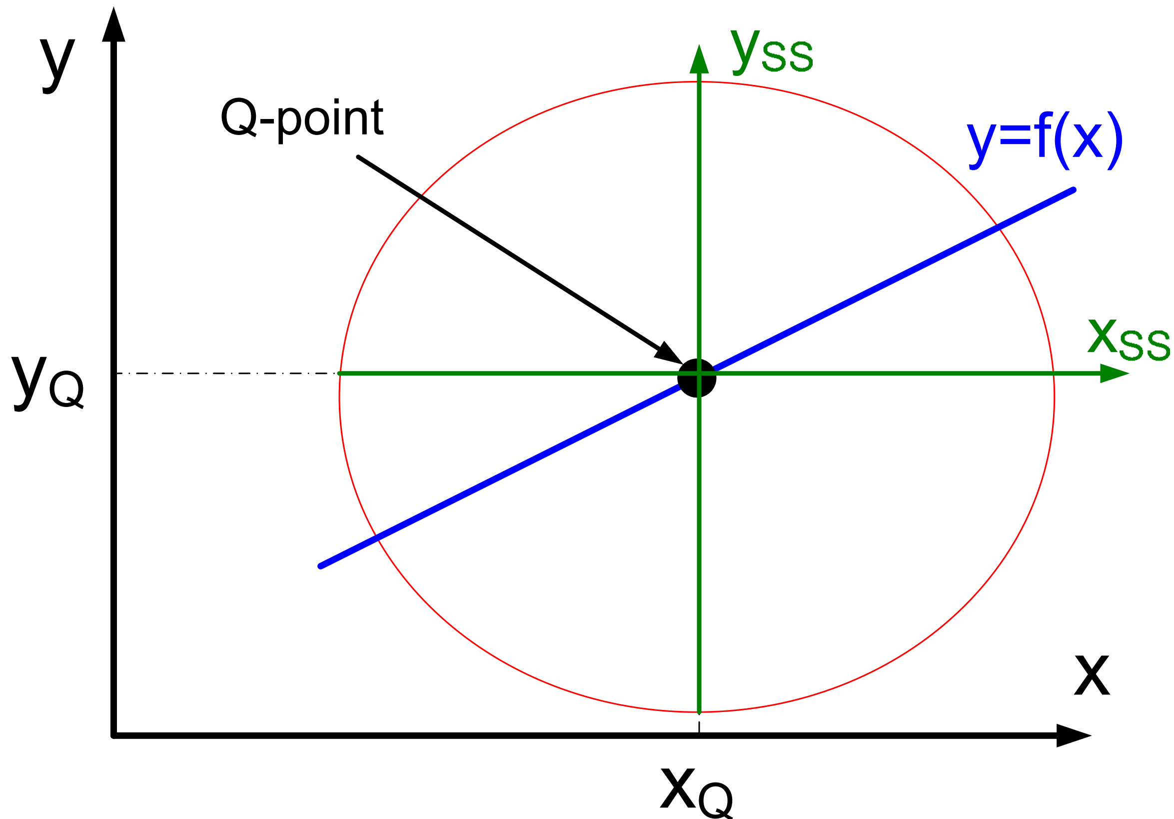


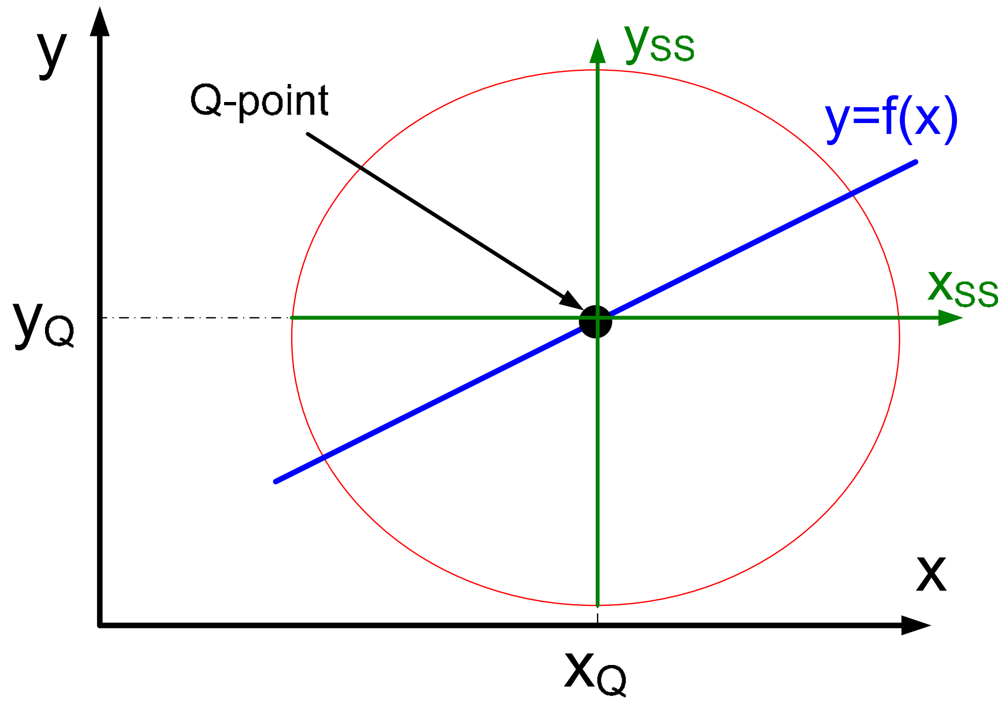
Changing coordinate systems:

$y_{SS}=y-y_Q$

$x_{SS}=x-x_Q$

$$y-y_Q=\left.\frac{\partial f}{\partial x}\right|_{x=x_Q}(x-x_Q) \longrightarrow y_{ss}=\left.\frac{\partial f}{\partial x}\right|_{x=x_Q} x_{ss}$$

Small-Signal Principle

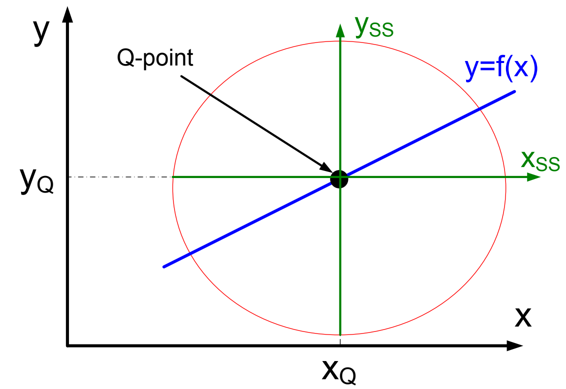


Small-Signal Model:

$$y_{ss} = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} x_{ss}$$

- *Linearized model for the nonlinear function y=f(x)*
- *Valid in the region of the Q-point*
- *Will show the small signal model is simply Taylor's series expansion at the Q-point truncated after first-order terms*

Small-Signal Principle



Observe:

$$y - y_Q = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} (x - x_Q)$$

$$y = y_Q + \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} (x - x_Q)$$

Small-Signal Model:

$$y = f(x_Q) + \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} (x - x_Q) \quad \longleftrightarrow \quad y_{ss} = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} x_{ss}$$

- *Mathematically, small signal model is simply Taylor's series expansion at the Q-point truncated after first-order terms*

Small-Signal Principle

Goal with small signal model is to predict performance of circuit or device in the vicinity of an operating point

Operating point is often termed Q-point

Will be extended to functions of two and three variables

Small-signal Operation of Nonlinear Circuits

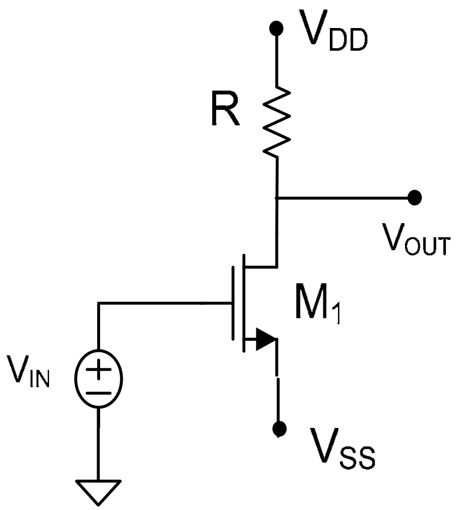
- Small-signal principles

Example Circuit

- Small-Signal Models

- Small-Signal Analysis of Nonlinear Circuits

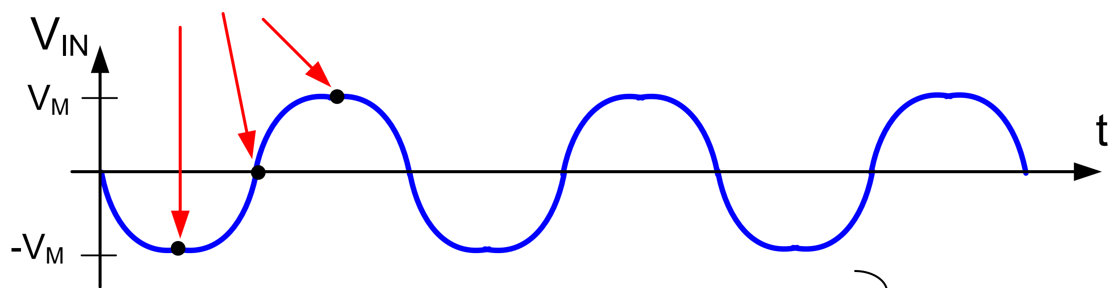
Small signal analysis example



By selecting appropriate value of V_{SS}, M_1 will operate in the saturation region

Assume M_1 operating in saturation region

Consider three points on the input waveform



$V_{IN}=V_M \sin\omega t$

V_M is small

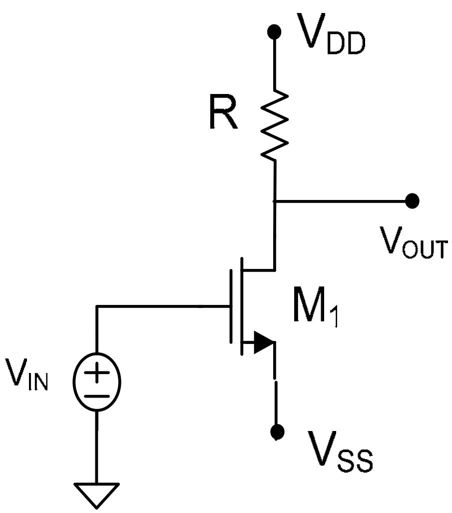
$$I_D = \frac{\mu C_{ox} W}{2L}\left(V_{IN}-V_{SS}-V_T\right)^2$$

$$V_{OUT}=V_{DD}-I_D R$$

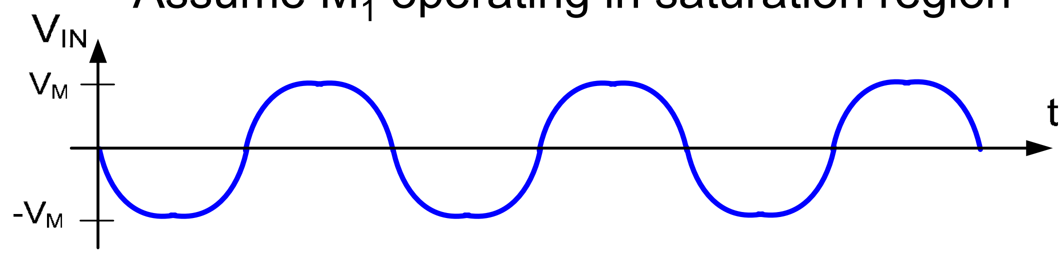
$$V_{OUT} = V_{DD} - \frac{\mu C_{ox} W}{2L}\left(V_{IN}-V_{SS}-V_T\right)^2 R$$

Small signal analysis example

By selecting appropriate value of V_{SS}, M_1 will operate in the saturation region



Assume M_1 operating in saturation region



$$\left.\begin{aligned} V_{OUT} &= V_{DD} - I_D R \\ I_D &= \frac{\mu C_{OX} W}{2L}\left(V_{IN} - V_{SS} - V_T\right)^2 \end{aligned}\right\}$$

$V_{IN} = V_M \sin\omega t$

V_M is small

$$\mathbf{I_{DQ}} = \frac{\boldsymbol{\mu}\,\mathbf{C_{OX}}\,\mathbf{W}}{\mathbf{2L}}\left(\mathbf{V_{SS}} + \mathbf{V_T}\right)^2$$

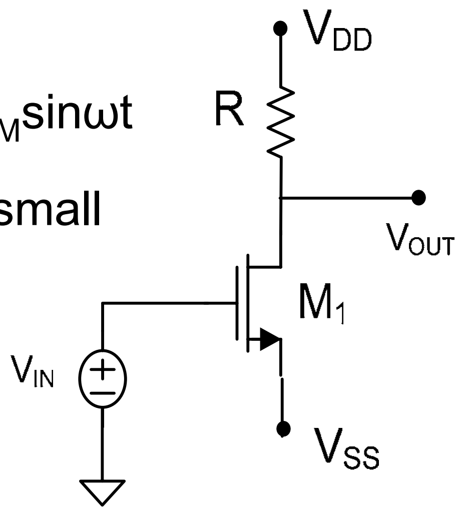
$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_{IN} - V_{SS} - V_T\right)^2 R$$

$$\mathbf{V_{OUT}} = \mathbf{V_{DD}} - \frac{\boldsymbol{\mu}\mathbf{C_{OX}}\mathbf{W}}{\mathbf{2L}}\left(\mathbf{V_M}\sin\omega\mathbf{t} - \left[\mathbf{V_{SS}} + \mathbf{V_T}\right]\right)^2 \mathbf{R}$$

Small signal analysis example

$V_{IN}=V_M\sin\omega t$

V_M is small



$$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}\left(V_M\sin\omega t-[V_{SS}+V_T]\right)^2R$$

$$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}[V_{SS}+V_T]^2\left(1-\frac{V_M\sin\omega t}{[V_{SS}+V_T]}\right)^2R$$

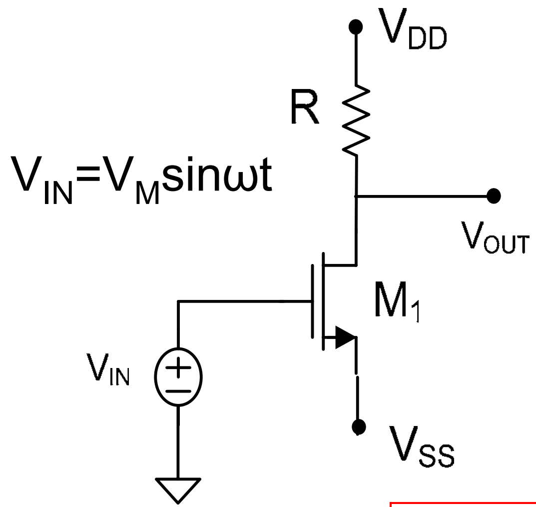
Recall that is x is small $\quad (1+x)^2\cong 1+2x$

$$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}[V_{SS}+V_T]^2\left(1-\frac{2V_M\sin\omega t}{[V_{SS}+V_T]}\right)R$$

$$V_{OUT}=\left\{V_{DD}-\frac{\mu C_{OX}W}{2L}[V_{SS}+V_T]^2R\right\}-\frac{\mu C_{OX}W}{2L}[V_{SS}+V_T]^2\left(\frac{2V_M\sin\omega t}{[V_{SS}+V_T]}\right)R$$

$$V_{OUT}=\left\{V_{DD}-\frac{\mu C_{OX}W}{2L}[V_{SS}+V_T]^2R\right\}-\left\{\frac{\mu C_{OX}W}{L}[V_{SS}+V_T]R\right\}V_M\sin\omega t$$

Small signal analysis example



$$V_{OUT} = \left\{ V_{DD} - \frac{\mu C_{OX} W}{2L} \left[V_{SS} + V_T \right]^2 R \right\} - \left\{ \frac{\mu C_{OX} W}{L} \left[V_{SS} + V_T \right] R \right\} V_M \sin \omega t$$

Quiescent Output

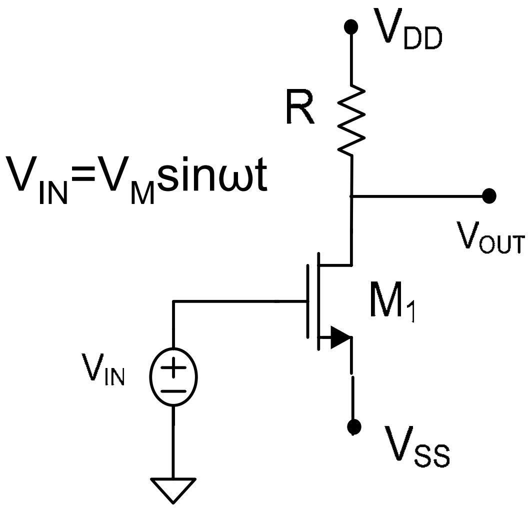
ss Voltage Gain

$$A_v = -\frac{\mu C_{OX} W}{L} \left[V_{SS} + V_T \right] R$$

Alternately, substituting from the expression for I_{DQ} we obtain

$$A_v = -\frac{2 I_{DQ} R}{\left[V_{SS} + V_T \right]}$$

Small signal analysis example



$$A_v = -\frac{2 I_{DQ} R}{\left[V_{SS} + V_T\right]}$$

Observe the small signal voltage gain is twice the Quiescent voltage across R divided by $V_{SS}+V_T$

- This analysis which required linearization of a nonlinear output voltage is quite tedious.
- This approach becomes unwieldy for even slightly more complicated circuits
- A much easier approach based upon the development of small signal models will provide the same results, provide more insight into both analysis and design, and result in a dramatic reduction in computational requirements

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- Example Circuit
- Small-Signal Models
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